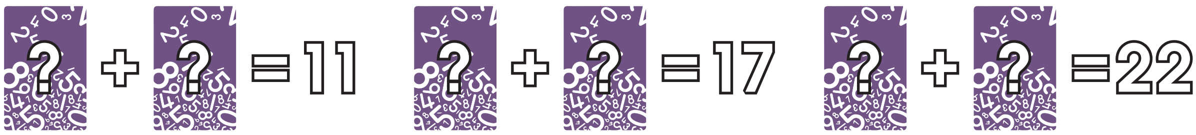
Problem of the Week #11

Three Card Total

11

Willow has three blank cards. She wrote some numbers on them:

When she placed the cards in pairs the following totals could be made:



What was written on the cards?

Problem of the Week #12

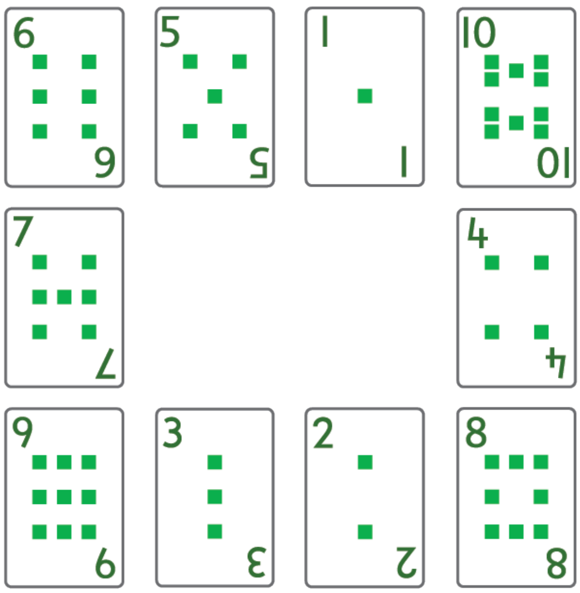
Card Rectangle

12

These cards add up to 22 in every row and column.

How many more arrangements can you make where all the sides add up to **22**?

Note: Swapping middle cards (e.g. 5 and 1 to be 1 & 5) doesn't count as a new arrangement



Teachers Notes:

Problem 11:

3, 8, 14.

Adding 11, 17 & 22 gives 50. Here all cards are counted twice (as they appear in two equations each).

To count each card only once we divide by two. 50 ÷ 2 = 25.

Therefore the sum of the three cards is 25.

One of the equations states that two cards added sum to 22, so the remaining card is 3.

Then 3 + _ = 11, so one of the cards is 8.

Then 3 + _ = 17, so the other card is 14.

Problem 12:

There are three more (making four in total) basic arrangements that sum to 22, each with 4 variations of where the cards are swapped around inside the rows.

How to check your arrangement: The sum of 1 to 10 = 55. Add the corner numbers again as they appear twice (in both a row and a column). If that total is divisible by 4 it is a valid rectangle.

6	5	1	10
7			4
9	3	2	8

6	1	5	10
7			4
9	3	2	8

6	5	1	10
7			4
9	2	3	8

6	1	5	10
7			4
9	2	3	8

6	3	5	8
7			4
9	1	2	10

6	5	3	8
7			4
9	1	2	10

6	3	5	8
7			4
9	2	1	10

6	5	3	8
7			4
9	2	1	10

8	2	3	9
4			7
10	1	5	6

8	3	2	9
4			7
10	5	1	6

8	2	3	9
4			7
10	1	5	6

8	2	3	9
4			7
10	5	1	6

8	5	3	6
4			7
10	1	2	9

8	5	3	6
4			7
10	2	1	9

8	3	5	6
4			7
10	1	2	9

8	3	5	6
4			7
10	2	1	9

Extending the problem solving to find rectangles with different sums:

The **smallest** numbers you could possibly have in the corners are 1, 2, 3 & 4. But this won't work (try it if you'd like). 55 (sum of the cards) + 1 + 2 + 3 + 4 = 65.

We know our rectangle will not work unless the total is divisible by four. The lowest set of corner numbers that will work is when the corners sum up to 13 as 55 + 13 = 68.

On the upper bound, the highest numbers we could possibly try is 7, 8, 9 & 10. This sums to 34. But 55 + 34 = 89. Eighty nine is not divisible by four. 88 is divisible by four however, so the highest sum of the corners we can use is 33.

Corner sums of 13 and 33 are known to work. Therefore every interval of 4 inside that range will also work. This means the corner sums that produce valid rectangles are 13, 17, 21, 25, 29 & 33. The line totals for those rectangles will be 17, 18, 19, 20, 21 & 22.

55 + 13 (corners) = 68. Line total of 68 ÷ 4 = 17

55 + 17 (corners) = 72. Line total of 72 ÷ 4 = 18

55 + 21 (corners) = 76. Line total of 76 ÷ 4 = 19 etc.