THE BEST OF

MATH MATH WORLD

1992 - 1996

OVER 50 REPRODUCIBLE ACTIVITIES

WITH ANSWERS

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The Mathematical Association of Western Australia Inc.

MAΣWA

Contents

Acknowledgements

Math Math World is produced by the Mathematical Association of Western Australia and the Australian Association of Mathematics Teachers for the enjoyment of upper primary and lower secondary students across Australia.

Numerous requests have been received for back issues of math Math World and answers to the various activities. The Best of Math Math World 1992 - 1996 is an attempt to put together the most popular activities for the five year period from 1992 - 1996.

The support of MAWA and AAMT is gratefully acknowledged along with the work of previous editors of It's a Math Math World which was the West Australian forerunner to the National Math Math World.

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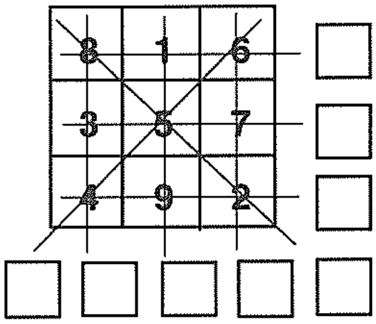
Typesetting: J. Swan, L. Swan and D. Swan

1 Magic Squares

Magic Squares were used in China as early as 2000 B.C. and were introduced into Europe during the Fifteenth Century.

Examine the number square below. Add the numbers in each row. Add the numbers in each column. Add the numbers in each diagonal.



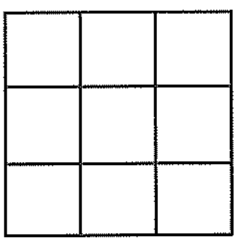
Each row, column and diagonal in a Magic Square adds up to the same amount —in this case 15.

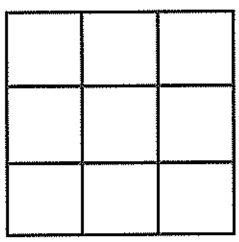
There are many different "Magic Squares" i e, where all rows, columns and diagonals add to the same number. See how many you can make using the numbers from 1 to 9. You may like to cut out the numbers at the bottom of the page and use the blank magic squares.

Remember to record your answers.

Every "Magic Square" has eight different rotations and reflections. Try to find the other seven for the magic square shown above.

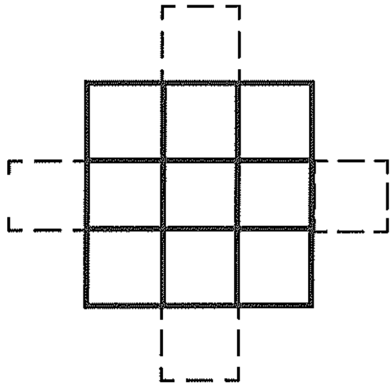
1 2 3 4 5 6 7 8 9





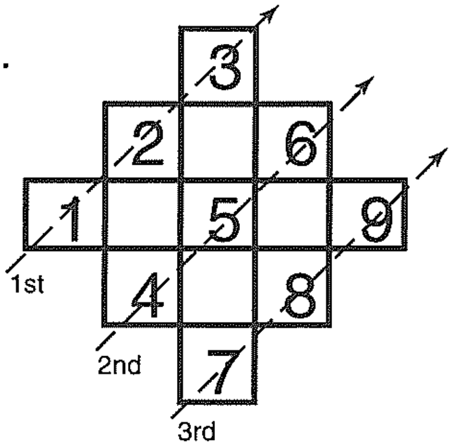
2 Making Magic Squares

To construct a Magic Square of order 3, try the following procedure. Draw an array of 9 cells and add 4 temporary cells as shown in the diagram on the right.



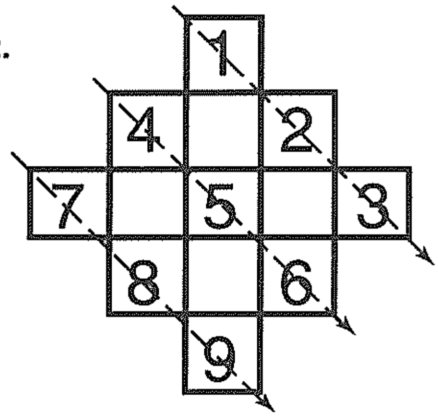
Write the starting number in any one of the temporary cells and then work in a diagonal pattern to fill in five of the nine cells.

1.



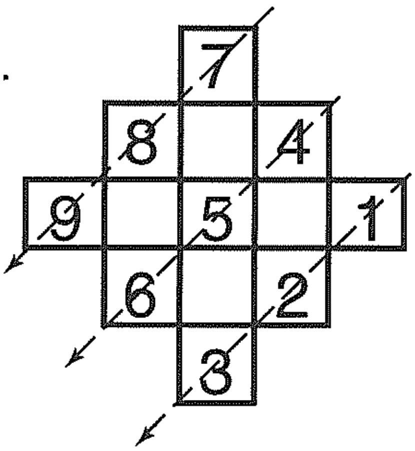
2.

OR



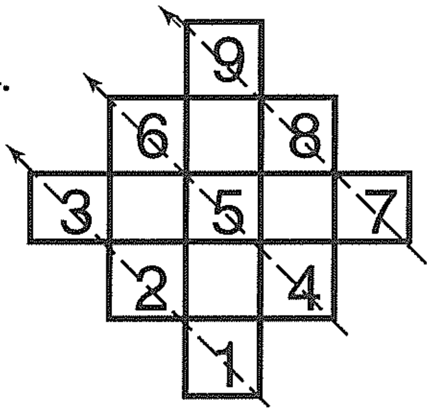
3.

OR



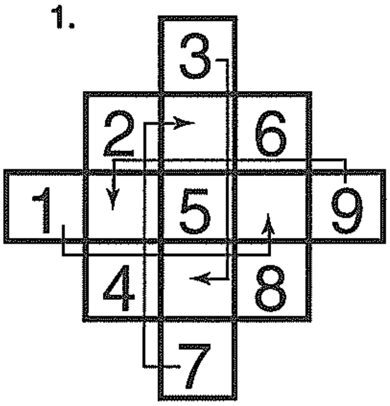
4.

OR



The remaining cells are filled in using the following pattern:

1.



→

2	7	6
9	5	1
4	3	8

Complete the other three Magic Squares by following the pattern shown above.

3 Reflecting Magic Squares

Every Magic Square has eight rotations and reflections. In activity 2 we created four Magic Squares using the numbers from 1 to 9. A further four Magic Squares may be created by reflection.

2	7	6
9	5	1
4	3	8

6	7	2
1	5	9
8	3	4

Use the reflection technique shown above to create three more Magic Squares from the remaining three Magic Squares created in Activity 2.

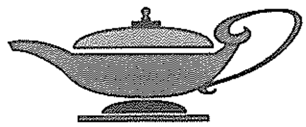
Look at the eight Magic Squares you have created. What do you notice about the middle number in each of the squares?

Try to find a relationship between the centre number and the total for each row, column and diagonal.

4 Magic Square Investigation

- Double each number in one of the magic squares you have created. Now try adding the numbers in the rows, columns and diagonals. What do you notice?
- Do all the rows, columns and diagonals still add to the same amount?
- Add five to each number in a magic square and note what happens.
- Predict what will happen if you add ten to each number.
- Verify your prediction by trying some.

Investigate what happens if you:



- Subtract a constant (eg 2, 5, 10)
- Multiply by a constant.

5 Even-order Magic Squares

In the previous activities we produced odd-order magic squares i.e. 3 x 3. A simple way of constructing a 4 x 4 (even-order) magic square is shown below.

Write down the numbers from 1 to 16 in each of the 16 cells of the 4 x 4 square. Then draw an asterisk in the centre of the square.

1	2	3	4
5	6	7	8
9	10	11	12
13	14	15	16

1	2	3	4
5	6	7	8
9	10	11	12
13	14	15	16

Swap the numbers at either end of each point of the asterisk, to produce a magic square.

1	15	14	4
12	6	7	9
8	10	11	5
13	3	2	16

- What constant is formed each time a row, column or diagonal is added?
- Use a different set of sixteen numbers to produce a new even-order magic square.
- Try using the rotation and reflection technique shown in Activity 3 to create a 4 x 4 magic square with the same constant (34) as the original magic square.
- Use the technique of adding or multiplying constants as shown in previous activities to create some new magic squares. Check that they are truly magic squares by adding the rows, columns and diagonals to see if the same constant is produced.

6 Magic from the Calendar

The Calendar provides an excellent starting point for producing a 4 x 4 magic square.

JANUARY

S	M	Tu	W	Th	F	S
1	2	3	4	5	6	7
8	9	10	11	12	13	14
15	16	17	18	19	20	21
22	23	24	25	26	27	28
29	30	31				

FEBRUARY

S	M	Tu	W	Th	F	S
			1	2	3	4
5	6	7	8	9	10	11
12	13	14	15	16	17	18
19	20	21	22	23	24	25
26	27	28				

MARCH

S	M	Tu	W	Th	F	S
			1	2	3	4
5	6	7	8	9	10	11
12	13	14	15	16	17	18
19	20	21	22	23	24	25
26	27	28	29	30	31	

APRIL

S	M	Tu	W	Th	F	S
30						1
2	3	4	5	6	7	8
9	10	11	12	13	14	15
16	17	18	19	20	21	22
23	24	25	26	27	28	29

MAY

S	M	Tu	W	Th	F	S
	1	2	3	4	5	6
7	8	9	10	11	12	13
14	15	16	17	18	19	20
21	22	23	24	25	26	27
28	29	30	31			

JUNE

S	M	Tu	W	Th	F	S
				1	2	3
4	5	6	7	8	9	10
11	12	13	14	15	16	17
18	19	20	21	22	23	24
25	26	27	28	29	30	

JULY

S	M	Tu	W	Th	F	S
30	31					1
2	3	4	5	6	7	8
9	10	11	12	13	14	15
16	17	18	19	20	21	22
23	24	25	26	27	28	29

AUGUST

S	M	Tu	W	Th	F	S
		1	2	3	4	5
6	7	8	9	10	11	12
13	14	15	16	17	18	19
20	21	22	23	24	25	26
27	28	29	30	31		

SEPTEMBER

S	M	Tu	W	Th	F	S
					1	2
3	4	5	6	7	8	9
10	11	12	13	14	15	16
17	18	19	20	21	22	23
24	25	26	27	28	29	30

OCTOBER

S	M	Tu	W	Th	F	S
1	2	3	4	5	6	7
8	9	10	11	12	13	14
15	16	17	18	19	20	21
22	23	24	25	26	27	28
29	30	31				

NOVEMBER

S	M	Tu	W	Th	F	S
			1	2	3	4
5	6	7	8	9	10	11
12	13	14	15	16	17	18
19	20	21	22	23	24	25
26	27	28	29	30		

DECEMBER

S	M	Tu	W	Th	F	S
31					1	2
3	4	5	6	7	8	9
10	11	12	13	14	15	16
17	18	19	20	21	22	23
24	25	26	27	28	29	30

Choose a 4 x 4 block of dates from any month.

e.g.

S	M	Tu	W	Th	F	S
1	2	3	4	5	6	7
8	9	10	11	12	13	14
15	16	17	18	19	20	21
22	23	24	25	26	27	28
29	30	31				

3	4	5	6
10	11	12	13
17	18	19	20
24	25	26	27

Now use the method shown in activity 5 to create a magic square. Check the result by adding all the rows, columns and diagonals.

7 Magic Formula

Magic squares may be produced using the formula below, where a, b, c, d, w, x, y, z represent eight different numbers.

Substituting the numbers:

a = 2, b = 5, c = 3, d = 10
w = 1, x = 6, y = 9, z = 11

a + w	d + y	b + z	c + x
c + z	b + x	d + w	a + y
d + x	a + z	c + y	b + w
b + y	c + w	a + x	d + z

we get:

3	19	16	9
14	11	11	11
16	13	12	6
14	4	8	21

Check that it really is a magic square by adding all the rows, columns and diagonals. Make your own 4 x 4 magic square using this method. Try to find a quick way of determining the constant by using the original substituted numbers.

8 The Game of 15

The game of 15 is based on a 3 x 3 magic square.

Two players will need a 3 x 3 grid and a set of cards numbering from 1 to 9.

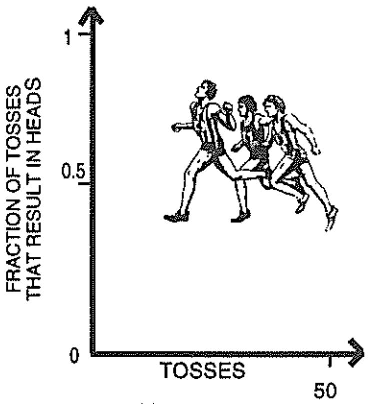
- ♠ One player has all the odd numbered cards and the other player all the even numbered cards.
- ♠ Players take turns laying a number on the grid until one player completes a row, column or diagonal that adds to 15.

1	2	3
4	5	6
7	8	9

9 In The Long Run

Toss a coin 50 times, recording the results in a table similar to the one shown below. Graph your results. *What do you notice happens to your line in the long run?*

OUTCOME	FRACTION HEADS	DECIMAL (2DP)
H	$^1/_1$	1
H	$^2/_2$	1
T	$^2/_3$	0·67
H	$^3/_4$	0·75
T	$^3/_5$	0·60
T	$^3/_6$	0·50



What do you think would happen if you plotted the fraction of tosses that resulted in tails on your graph?
Consider graphs of other class members – *are they similar to yours?*

10 Do or Die

Most games played with dice require that you throw a six with a single die before starting the game.

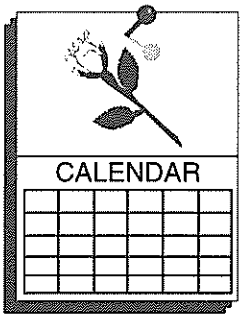
What is the most likely number of throws you think will have to be made before a six is rolled?

Experiment to find the answer.

Record the number of throws taken before a six turns up. Repeat ten times and average your results.

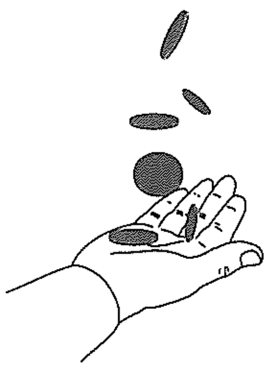
Surprised?

11 Baffling Birthdays



- What is the probability that two people have the same birthday?
- Carry out a survey of your class to determine birthdates.
- Are some months more popular than others?
- Why do you think this is the case?

12 Fair Go



Consider the following game between two players.

The first player tosses a coin. If it comes up a **head**, the first player wins. If a **tail** turns up the second player tosses the coin.

Should the second player's toss display a head the first player wins, a tail and the second player wins.

Obviously this game is **not fair**. Try to design a scoring system that makes the game fairer. Play the game for ten rounds using your scoring system and comment on your results.

13 DICE CRICKET

Rules:

1. Roll a die to see who bats and who bowls – the higher number chooses. Each player then writes the numbers 1 to 11 on to a score sheet.
2. The player batting rolls a die and scores the number of runs equal to the value displayed by the die, unless the RESULT is a five. A five is considered as an appeal for a wicket and the bowler is given the opportunity of rolling the die and determining if and how the one batting is out.

If the die turns up

a 1, the batter is out, hit wicket.

a 2, the batter is out, bowled.

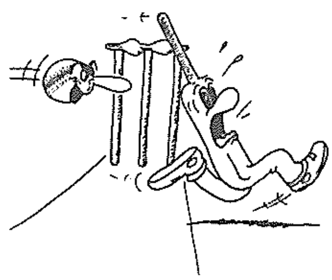
a 3, the batter is out caught.

a 4, the batter is out lbw.

a 5, the batter is not out.

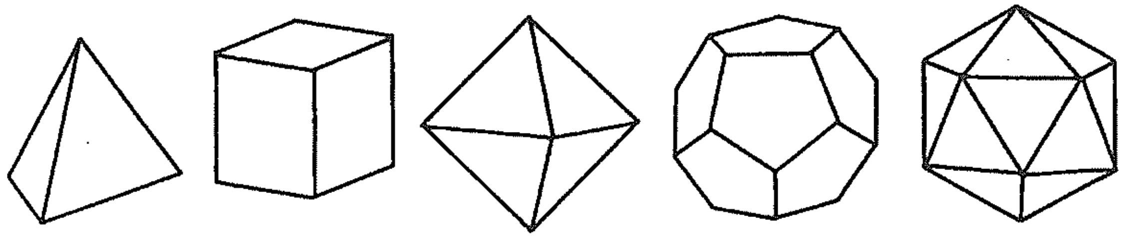
a 6, the batter is run out.

3. When a batter is out their score is tallied. The team batting continues until 10 team members are dismissed.
4. When the first team has been dismissed, the batting and bowling roles are reversed.
5. The 'Winner' is the team scoring the most runs.



14 Perplexing Polyhedra

THE TETRAHEDRON

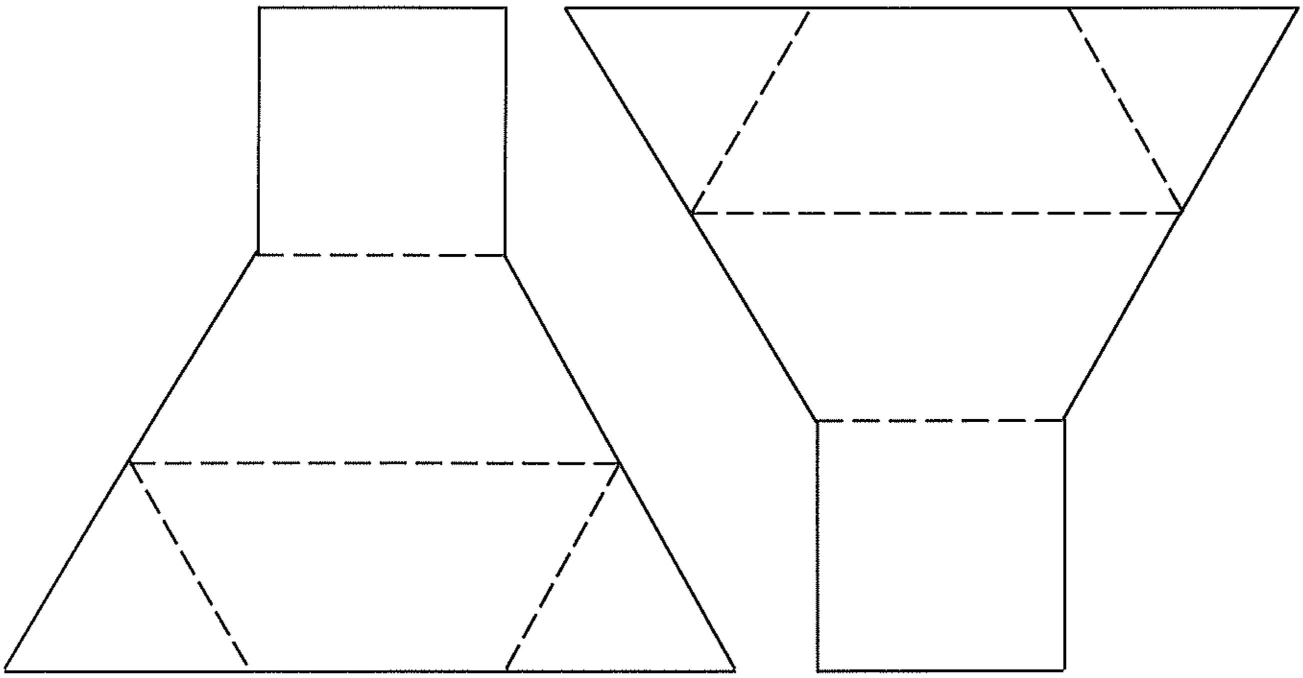


Tetrahedron Hexahedron (Cube) Octahedron Dodecahedron Icosahedron

Altogether there are five regular Polyhedra. The simplest regular polyhedron is the tetrahedron. A tetrahedron is made up of 4 equilateral triangles and forms a pyramid.

Here is a puzzle for you to try that involves using the shape below to produce a tetrahedron.

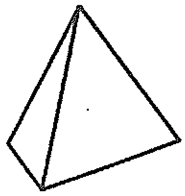
Make two copies of the figure shown on light card and cut them out. Fold each one along the dotted lines and join them together so that two identical shapes are formed. Now try to put these two shapes together to form a terahedron.



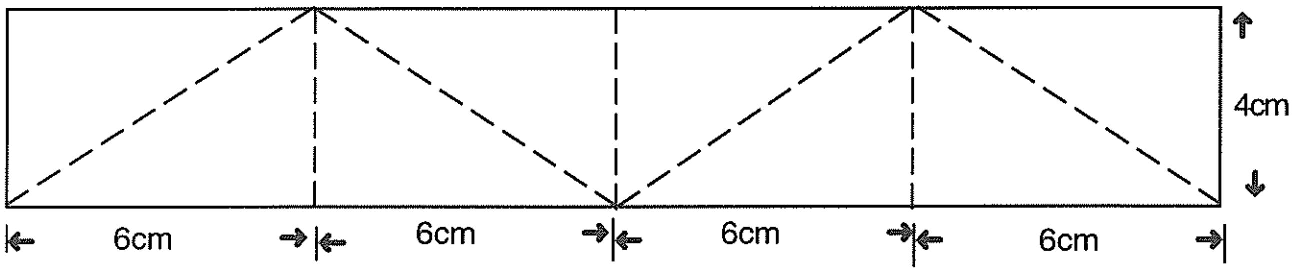
> The five regular polyhedrons are sometimes called the Platonic Solids, after the Greek philosopher Plato.

15 Frenzied Folding

THE TETRAHEDRON



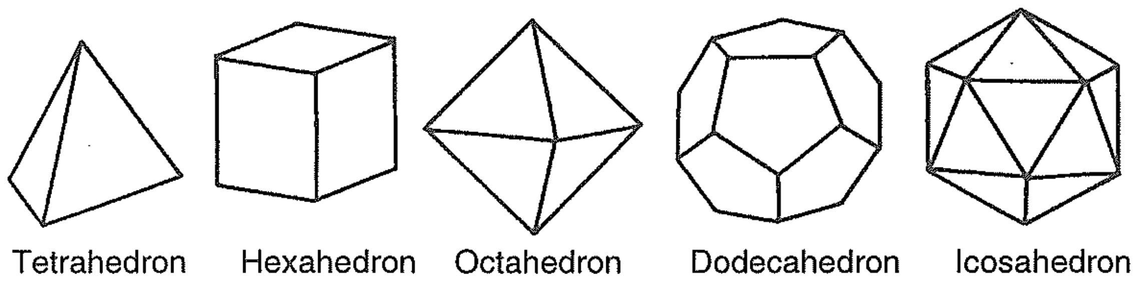
You will need a piece of light card 24 cm x 4 cm.
Divide the long rectangle into four smaller rectangles and mark in the diagonals as shown. (Note: Diagram is not to scale.)



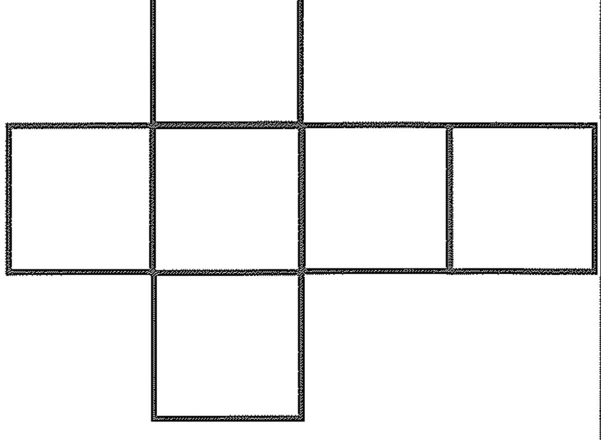
Crease along all the dotted lines. Join the two ends so that a band is formed.
Fold to produce a tetrahedron.

16 Perplexing Polyhedra

THE HEXAHEDRON



The most familiar regular polyhedron is the Cube or Hexahedron. The cube is made up of six squares. A net that produces a cube is shown at the right.



How many different nets can you make that fold up to produce a cube?

You may wish to use cardboard squares to help you.

Remember to record your answers.

17 Codes & Ciphers

DPEFE NFTTBHFT DBo CF WFSZ DPOGVTJOH

The message above is written in code. Each letter in the message has been replaced by the letter immediately following it in the alphabet. This technique was used by Julius Caesar in his campaigns and is probably the simplest method of enciphering.

To decipher this message, list the letters of the alphabet down the page, then next to that list, place a list that goes Z, A, B, C...Y so that Z is next to A, A is next to B etc. ... Find the code letter in the first list, and the solution will be next to it in the second list.

A	Z
B	A
C	B
D	C
E	D

Try coding some messages yourself using this technique.

18 Polybius Code

The following code is based on the substitution cipher developed by Polybius in the second century before Christ.

	1	2	3	4	5
1	A	B	C	D	E
2	F	G	H	I	J
3	K	L	M	N	O
4	P	Q	R	S	T
5	U	V	W	X	YZ

Unlike reading co-ordinates, the following letters are coded by noting the vertical number followed by the horizontal number. The letter "J" would be coded as 25 and MATH MATH WORLD would be coded as

33.11.45.23.. 33.1.45. 23.. 53. 35. 43. 32.14

Decode this message:

24.. 32. 35. 52. 15.. 33. 11. 45. 23. 44

Code your own messages using the Polybius square above.

Develop your own Polybius square, starting with A in the bottom left part of the square (i.e. at the position 51) and use it to code your own messages

19 Codes & Ciphers II

A simple cipher may be developed using squares or rectangles. For example the sentence
"MATHS IS MY FAVOURITE SUBJECT"
consists of 25 letters. You may recognise 25 as being a square number (i.e. $5^2 = 25$).
Draw a 5 x 5 box on graph paper and then write the above sentence in the square.

M	A	T	H	S
I	S	M	Y	F
A	V	O	U	R
I	T	E	S	U
B	J	E	C	T

Now write the sentence reading downwards rather than across.

MIAIB ASVTJ TMOEE HYUSC SFRUT

Try decoding the following message:

AAMCE LTASA LHTIS MEISY

(*HINT You can see that this message was originally encoded on a 5 x 4 grid because there are 4 words containing 5 letters.*)

Try some more of your own.

20 Codes & Ciphers – ISBN

If you look on the cover of most books you will find an **ISBN** or **International Standard Book Number**.

An ISBN consists of 10 digits e.g. **ISBN 0 9588575 5**

ISBN	**0**	**9588575**	**5**
	Nation/Language	Publisher & Title	Check Digit

The first digit identifies the nation and language. This is followed by a number which identifies the publisher and book title. The last digit is called a check digit.

Checking the check digit.

A check digit is used to determine whether the ISBN is entered correctly. Follow these steps to check the check digit.

e.g. ISBN	**0**	**9**	**5**	**8**	**8**	**5**	**7**	**5**	**5**
	a	**b**	**c**	**d**	**e**	**f**	**g**	**h**	**i**

✍ Add $10a + 9b + 8c + 7d + 6e + 5f + 4g + 3h + 2i$

$$\mathbf{10 \times 0 + 9 \times 9 + 8 \times 5 + 7 \times 8 + 6 \times 8 + 5 \times 5 + 4 \times 7 + 3 \times 5 + 2 \times 5 = 303}$$

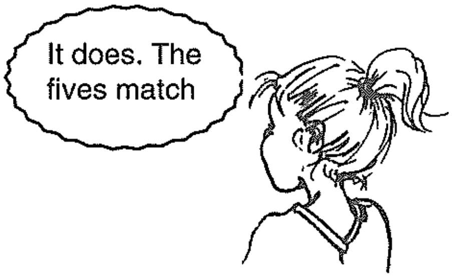
✍ Divide by 11 and ***find*** the remainder

$$\mathbf{303 \div 11 = 27 \text{ remainder } 6}$$

✍ Subtract the remainder from 11

$$\mathbf{11 - 6 = 5}$$

This should produce the check digit.



Try these:

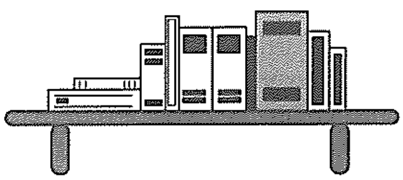
0 9588575 4 7

0 9588575 8 x

0 73164567 7

0 64602382 9

What does x represent?



21 Codes & Ciphers — The Binary System

Our Hindu-Arabic numeration system is a base ten system, i.e. we have ten digits which we use in conjunction with place value to form all the numbers we need.

Not all Numeration systems are base ten. The Ancient Babylonians used a sexagesimal, or base sixty system, whilst the Maya used a vigesimal or base 20 system.

Computers and calculators use a base two system called the Binary System (bi meaning two). In a Binary system only two digits are used, 0 and 1.

16	8	4	2	1	DECIMAL NUMBERS
				1	1
			1	0	2
			1	1	3
		1	0	0	4
		1	0	1	5
		1	1	0	6
		1	1	1	7
	1	0	0	0	8
	1	0	0	1	9

- Try to write ten using Binary notation.
- Write the following numbers in Binary notation

  12, 15, 19, 25, 28, 31.
- What do the following numbers represent?

  110010, 111111, 1000001,

  1100100, 1011010.
- A table similar to the one shown at the left may help.

22 Codes & Ciphers — Bar Codes

9 310062 540156

Most products are labelled with a barcode. A barcode consists of vertical bars. The bars and the white spaces between represent the digits 0 and 1, which may be used to form binary numbers. Each product is given a number. A check digit is used to determine whether the code is correct. To check the following barcode use these steps.

9 310062 540156

- Start with the digit to the left of the check digit and then add every second digit.

  e.g. 5 + 0+ 5 + 6 + 0 + 3 = 19
- Multiply the result by three 3 x 19 = 57
- Add the remaining digits 1 + 4 + 2 + 0 + 1 + 9 = 17
- Add the results from steps 2 and 3 57 + 17 = 74
- The check digit should be the smallest number that may be added to the result of step 4 to make a multiple of ten.

  e.g. 74 + 6 = 80.
- The check digit should be 6.

Use the steps outlined above to check the following barcodes

9 312345 678907   4 006381 114615

9 310353 080101   9 312650 901905 .

23 Patterns in the Hundred Square

Draw a 3 x 3 square anywhere on the hundreds chart.

15	16	17
25	26	27
35	36	37

- Add the four corner numbers together, e.g. 15 + 17 + 37 + 35 = 104
- Try to find a quick way of arriving at this total.
- Try to find a relationship between the centre number and the sum of the four corner numbers. Write down this relationship.
- Try exploring for different positions on the chart. Does your rule still work?
- Try using other odd sized squares, e.g. 5 x 5 and 7 x 7. Does your rule still work?

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

24 Multiple Patterns in the Hundred Square

Several patterns may be found by colouring in various multiples on the hundreds chart. Try colouring in the multiples of nine and then eleven. Write about what you notice. Repeat using a 0 – 99 chart.

- Now try shading some other multiples e.g. 2, 3, 4, 5, 6, 7, 8 and 12.
- Write about any patterns you notice.
- Let's return to the multiples of nine. What happens if we start at 6 and then colour every ninth square?
- You are probably already familiar with the digit pattern in the nine times table, i.e. if you add the digits in a number divisible by nine you will always eventually equal 9,
  e.g. $54 \rightarrow 5 + 4 \rightarrow 9$,
  $99 \rightarrow 9 + 9 \rightarrow 18 \rightarrow 1 + 8 = 9$.
  Consider the digit sums for 6, 15, 24 etc. What do you notice?
- Do similar patterns exist when you constantly add nine to other numbers?

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

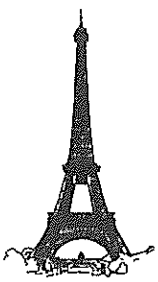
25 Patterns in the Hundred Square II

- Draw a rectangle on a hundred's chart.
- Add each pair of numbers in opposite corners e.g. 22 + 46 = ? 26 + 42 = ?
- What do you notice about the totals?
- Try other rectangles, different sizes, different positions. Write about what you notice.
- Which other pairs of numbers inside the rectangle add up to the same amount as the opposite corner pairs?
- Try to find a quick way of working out the sum of all the numbers in any 5 x 3 rectangle.
- Try other shapes: squares, rhombus and parallelograms.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

26 Patterns in Pascal's Triangle

The pattern of numbers shown below is named after a French mathematician, Blaise Pascal, who lived in the 17th century. Pascal however, was not the first to notice this pattern. It appears that the triangle was in existence around 200 BC.



1

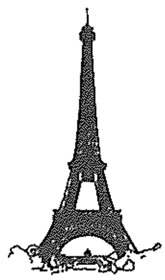
1 1

1 2 1

1 3 3 1

1 4 6 4 1

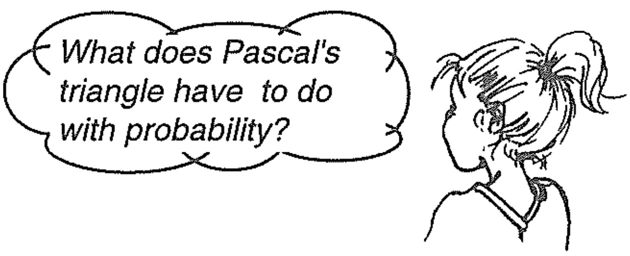
1 5 10 10 5 1



- Look along the diagonals.
- Write down any patterns that you notice.
- Try to explain how each new row is formed.
- Continue the triangle by adding three more rows.
- Add the numbers in each row.
- What do you notice?

27 Pascal's Triangle and Probability

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1

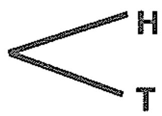


If a coin is tossed there are two possible results – either the coin turns up **heads** or **tails.**

We can write this as **P (1 head) = 1/2**

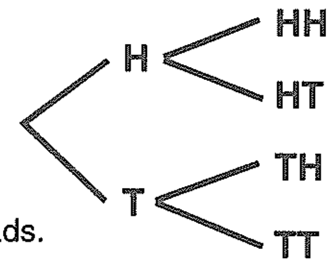
P (0 heads) = 1/2

and show this on a diagram



When a coin is tossed twice there are 4 possible outcomes.

These are shown on the diagram at the right.



- Write down the chance of tossing: 2 heads, 1 head, 0 heads.
- A coin is tossed three times. What is the chance of tossing:

  3 heads, 2 heads, 1 head, 0 heads.
- Now look back at Pascal's triangle. Write down what you notice about the rows of Pascal's Triangle and the chances of getting a head for various numbers of coin tosses.
- Predict the chances of getting 5 heads, 4 heads, 3 heads, 2 heads, 1 head or 0 heads if a coin is tossed 5 times. Pascal's Triangle should help you!

Challenge

- Find the value of 11^2, 11^3. What do you notice when you look at the rows of Pascal's Triangle?
- Predict the value of 11^4.
- Now try 11^5

28 Patterns in Pascal's Triangle II

1

1 1

1 2 1

1 3 3 1

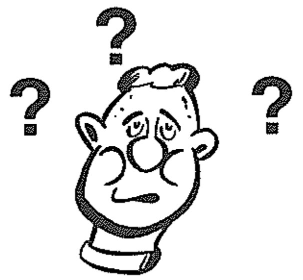
1 4 6 4 1

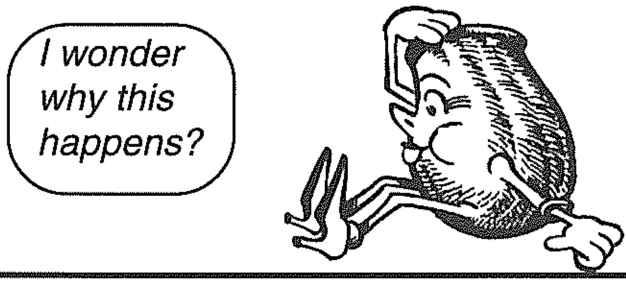
1 5 10 10 5 1

- Choose a number in the triangle e.g. **4**
- Multiply the six numbers surrounding this number

  1 x 1 x 5 x 10 x 6 x 3 = 900
- Find the square root of this number

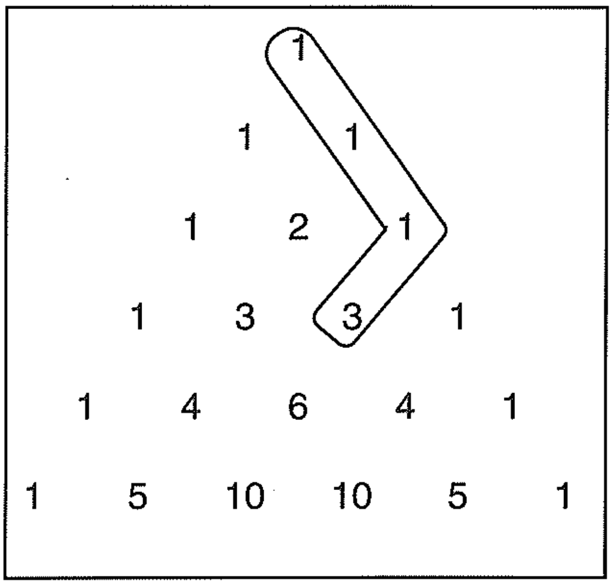
  $\sqrt{900} = 30$
- Try the same procedure for several other numbers in the triangle
- Write about what you notice.

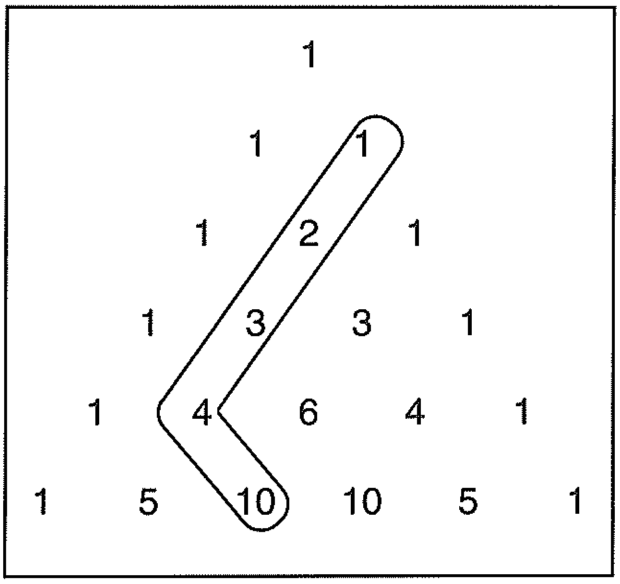




29 Pascal's Stocking Pattern

The sum of any series of numbers running obliquely in the triangle is found in the "foot" of the stocking.
This is shown in the diagram below eg 1 + 1 + 1 = 3 and 1 + 2 + 3 + 4 = 10.



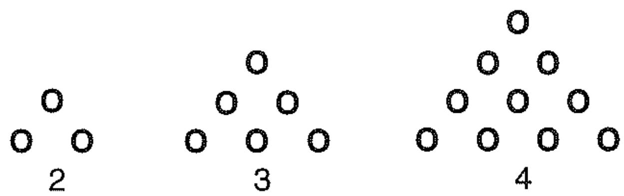


Try some of your own 'stocking sums'. You may wish to test your theory on a larger version of Pascal's Triangle

30 Figurate Numbers

Consider the following diagonal in Pascal's Triangle.

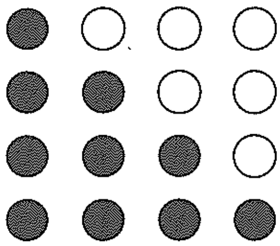
This set of numbers is known as the Triangular Numbers. They are called triangular numbers because of the geometric pattern they form.



1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1

Note the pattern formed as each new row is added.

- ✎ Find the first 10 triangular numbers.
- ✎ Try adding any two consecutive triangular numbers. What happens?
- ✎ Try other consecutive triangular numbers.
- ✎ Sixteen (a square number) is made up of two consecutive triangular numbers, 6 and 10.
- ✎ Can all square numbers be made this way?
- ✎ Try for all squares below 100 ie 4, 9, 16, 25, 36, 49, 64, 81.
- ✎ Does this always happen?
- ✎ Try to explain why it happens.



10 + 6 = 16

31 Fibonacci Numbers

Italian mathematician, Leonardo Fibonacci, discovered the following sequence of numbers.

1, 1, 2, 3, 5, 8, 13, 21, 34

Try to explain how each new number in the sequence is found.

- ✎ Add three more numbers to the sequence.

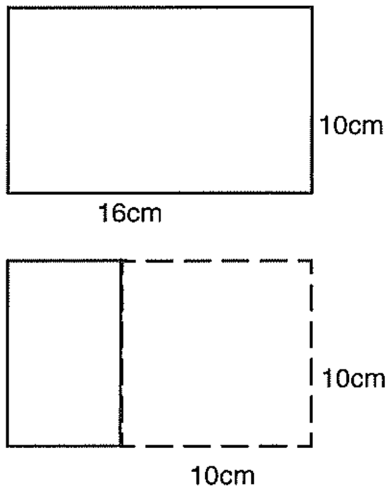
Some interesting patterns come to light when examining the sequence. For example

- ✎ Choose any three consecutive Fibonacci numbers **e.g. 5, 8, 13**
- ✎ Square the middle number. **8 x 8 = 64**
- ✎ Multiply the other two numbers. **5 x 13 = 65**
- ✎ Note the difference between the two results.
- ✎ Try some more consecutive numbers from the Fibonacci sequence and see what happens.

32 Golden Rectangles

The "Golden Rectangle" is well known in architecture. The rectangle conforms to the "Golden Ratio" which is 1 : 1·618. The ancient Greeks believed that the rectangles which conformed to this ratio produced the most pleasing shapes. The Egyptians also used the ratio in constructing the pyramids. Both the Greeks and the Egyptians felt that the ratio and the rectangle produced according to the ratio had magical powers.

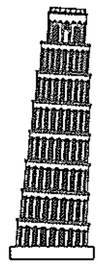
Try the following exercise and see what you think.



- Draw a golden rectangle of length 16 cm and width 10 cm.
- Cut off a square (10cm x 10cm)
- Measure the remaining piece.
  What do you notice?

- Try it again using the remaining piece.
- What do you notice?
- Try starting with a different Golden Rectangle
  e.g. 15 x 24 or 5 x 8.

33 Golden Fibonacci

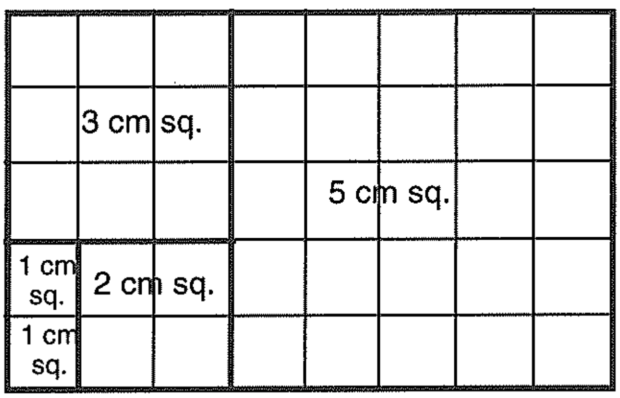


Fibonacci was an Italian mathematician who lived in the 12th and 13th centuries. He is famous for having developed a sequence of numbers, now named the Fibonacci Sequence.

1, 1, 2, 3, 5, 8, 13, 21, 34, 55.

The Fibonacci Sequences may be used to produce a golden rectangle.

- Use 1cm graph paper to cut out two 1cm squares. Then cut out a 2 cm square, then a 3 cm square, a 5 cm square and finally an 8 cm square.
- Join them together to form a rectangle.
- What are the length and width of the rectangle?
- Try adding a 13 x 13 square and then a 21 x 21 square.
- What happens?



34

Calendar Capers

- Choose any month from the calendar.
- Select four dates that form a 2 x 2 pattern.
- Draw a box around them.

January

S	M	T	W	T	F	S
	1	2	3	4	5	6
7	8	9	10	11	12	13
14	15	16	17	18	19	20
21	22	23	24	25	26	27
28	29	30	31			

February

S	M	T	W	T	F	S
				1	2	3
4	5	6	7	8	9	10
11	12	13	14	15	16	17
18	19	20	21	22	23	24
25	26	27	28	29		

March

S	M	T	W	T	F	S
31					1	2
3	4	5	6	7	8	9
10	11	12	13	14	15	16
17	18	19	20	21	22	23
24	25	26	27	28	29	30

April

S	M	T	W	T	F	S
	1	2	3	4	5	6
7	8	9	10	11	12	13
14	15	16	17	18	19	20
21	22	23	24	25	26	27
28	29	30				

May

S	M	T	W	T	F	S
			1	2	3	4
5	6	7	8	9	10	11
12	13	14	15	16	17	18
19	20	21	22	23	24	25
26	27	28	29	30	31	

June

S	M	T	W	T	F	S
30						1
2	3	4	5	6	7	8
9	10	11	12	13	14	15
16	17	18	19	20	21	22
23	24	25	26	27	28	29

July

S	M	T	W	T	F	S
	1	2	3	4	5	6
7	8	9	10	11	12	13
14	15	16	17	18	19	20
21	22	23	24	25	26	27
28	29	30	31			

August

S	M	T	W	T	F	S
				1	2	3
4	5	6	7	8	9	10
11	12	13	14	15	16	17
18	19	20	21	22	23	24
25	26	27	28	29	30	31

September

S	M	T	W	T	F	S
1	2	3	4	5	6	7
8	9	10	11	12	13	14
15	16	17	18	19	20	21
22	23	24	25	26	27	28
29	30					

October

S	M	T	W	T	F	S
		1	2	3	4	5
6	7	8	9	10	11	12
13	14	15	16	17	18	19
20	21	22	23	24	25	26
27	28	29	30	31		

November

S	M	T	W	T	F	S
					1	2
3	4	5	6	7	8	9
10	11	12	13	14	15	16
17	18	19	20	21	22	23
24	25	26	27	28	29	30

December

S	M	T	W	T	F	S
1	2	3	4	5	6	7
8	9	10	11	12	13	14
15	16	17	18	19	20	21
22	23	24	25	26	27	28
29	30	31				

- Add the four numbers together. $11 + 12 + 18 + 19 = 60$
- *Divide* the answer by four. $60 \div 4 = 15$
- Then *subtract* four from the answer. $15 - 4 = 11$
- Repeat this several times using different sets of dates, months and calendars.
- Write about what you notice.

35 Calendar Corners

- Choose any month from the calendar and draw a 3 x 3 box around nine dates.

NOVEMBER

S	M	T	W	T	F	S
					1	2
3	4	5	6	7	8	9
10	11	12	13	14	15	16
17	18	19	20	21	22	23
24	25	26	27	28	29	30

- Add the numbers in opposite corners.
- Try some other months.
- Write about what you notice.
- Vary the size of the rectangles and see what happens.
- Try to explain why this happens

36 Calendar Perimeter

- Choose a block of nine dates.
- Add all the dates on the perimeter of the block.

APRIL

S	M	T	W	T	F	S
	1	2	3	4	5	6
7	8	9	10	11	12	13
14	15	16	17	18	19	20
21	22	23	24	25	26	27
28	29	30				

- Divide the total by the centre number.
- Repeat several times using different blocks of dates from different months.
- Write about what you notice.

37 Calendar Count

Draw a 3 x 3 box around any nine dates from any month on the calendar.

	↓	↓	↓	
→	3	4	5	____
→	10	11	12	____
→	17	18	19	____
	____	____	____	

- Add the numbers in each row and in each column.
- Try several different sets of dates.
- What do you notice about the answers?
- Try to explain any relationships you notice

38 Calendar Cross

- Draw a 3 x 3 box around any nine dates from any month on the calendar.

DECEMBER

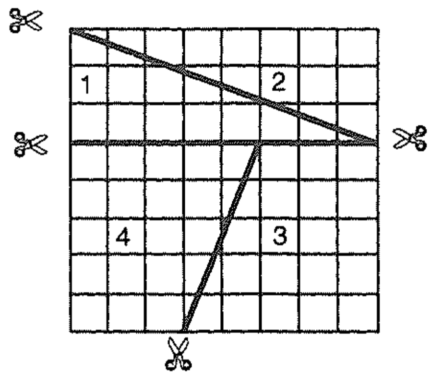
1	2	3	4	5	6	7
8	9	10	11	12	13	14
15	16	17	18	19	20	21
22	23	24	25	26	27	28
29	30					

- Draw in the diagonals.
- Add up the values along each diagonal.
- Try several examples.
- What have you found?
- Why do you think this occurs?

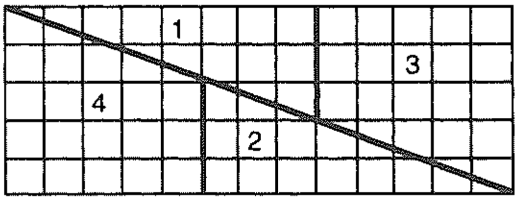
39 Missing Area

△ Mark out an 8 x 8 square as shown on a piece of graph paper

△ Find the area of the square.



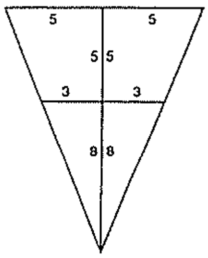
△ Cut out the square and re-form to make a rectangle. Now work out the area of the rectangle.

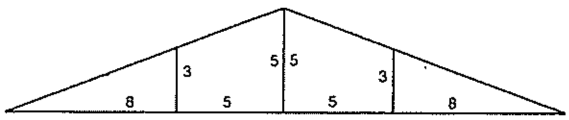


(Diagram not full size)

△ What has happened?

△ Two different isosceles triangles may be formed using the four pieces. Make each triangle.





△ Work out the area by using the formula:

Area of triangle = 1/2 b x h

△ Find the area of each triangle.

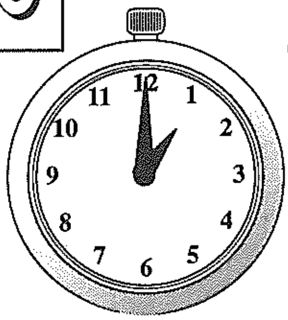
△ *What happens?*

△ Now form a trapezium with the four pieces and find the area. The area of a trapezium is found by averaging the lengths of the base and top of the trapezium and multiplying by the perpendicular height. Note what happens.

△ The four pieces may also be fitted to make a parallelogram. Find the area of the parallelogram.

△ *What has happened?*

40 Prime Time



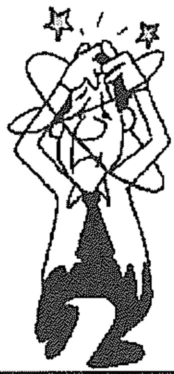
- Choose a prime number greater than 3 — e.g. 7
- Square the number — 49
- Add 15 — 64
- Divide by 12 and note the remainder — 5 r 4
- Repeat several times for different prime numbers.
- Write about what you notice.

41 Multiple Madness

- Choose any multiple of 7
  e.g. (5 x 7) 35
- Multiply by 15873
- Record the result.
- Try for other multiples of 7.
- What happens?
- Try to explain why it works.

42 Multiple Madness II

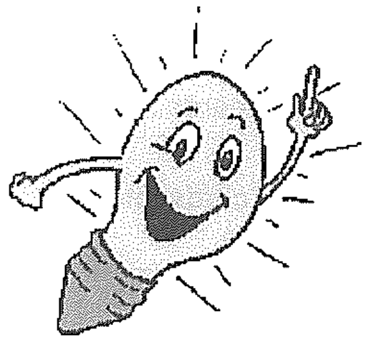
- Choose any multiple of 13
  e.g. (3 x 13) 39
- Multiply this number by 8547
- Record the result.
- Try other multiples of 13.
- What do you notice?
- Try to explain why it works.



43 Number Novelties

- Write down any 4-digit number — **e.g. 3146**
- Interchange the first and last digit of the number — **6143**
- Subtract the smaller number from the larger — **6143 − 3146 = 2997**
- Now interchange the first and last digits of the answer — **7992**
- Add this new number to the answer formed by the previous subtraction — **7992 + 2997**
- What is your final result?
- Try some more. What happens?
- Does this always work?
- What about 5, 6 or 7 digit numbers?

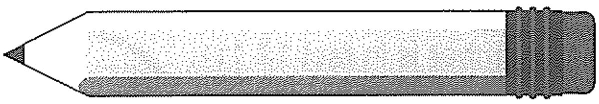
44 9 Guzinta



- Choose any whole number above 10 — **e.g. 43**
- Add the digits that make up the number — **4 + 3 = 7**
- Subtract this sum from the original number — **43 – 7 = 36**
- Divide the result by nine — **36 ÷ 9 =?**
- Try this several more times with different numbers.
- Write about what you notice.

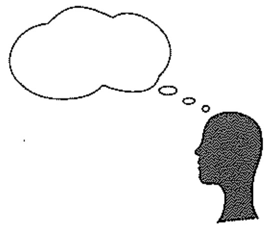
45 99 Guzinta

- Pick any 3-digit number where all the digits are different — **e.g. 417**
- Reverse the digits — **714**
- Subtract the smaller number from the larger number — **714 – 417 = 297**
- Divide the result by 99.
- Try this several more times.
- Write about what you notice.



46 Multiplication Shortcuts

- To square a two digit number ending in 5 — e.g. **45**
- Square the tens digit — **4 x 4 = 16**
- Add the tens digit to that product — **4 + 16 = 20**
- The last two digits will always be 25 so the answer is **2025**
- Try some other two digit numbers ending in 5
- Try to explain why this short-cut works.



47 Multiplication Shortcuts II

To square the "fifty numbers" — e.g. **56**

- The first two digits are found by adding 25 to the units digit — e.g. **25 + 6 = 31**
- The remaining two digits are found by squaring the units digit — e.g. **6 X 6 = 36**

Therefore the answer is **3136**

- Try squaring some other "fifty numbers" this way.
- Try to explain why this works.
- Try to develop a short-cut for squaring "forty numbers"

48 Mind Reader

- Ask a friend to write down any single digit number — e.g. **6**
- Multiply this digit by 10 — **60**
- Add the original digit — **66**
- Multiply the result by 9 — **594**
- Multiply by 11
- Then ask our friend to tell you the last digit in the result and offer to reveal the other three.

The other three digits are found this way

1. The ten's digit is one less than the units digit.
2. The hundred's digit is the difference between 9 and the unit's digit.
3. The thousand's digit is the difference between 9 and the ten's digit.

49 Magic Numbers

Here is a typical number trick which I am sure most students will recognise.Using a little Algebra, we see why the trick works.

- Think of a number — x
- Add 5 to your number — $x + 5$
- Multiply the result by 2 — $2x + 10$
- Subtract 4 — $2x + 6$
- Divide by 2 — $x + 3$
- Subtract your original number — $x + 3 - x$

Your answer is 3 — **3**

Try this second trick on someone and find out their age.

Ask the person to do this:

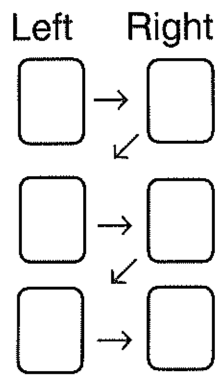
> *"Multiply your age by 3, add 6 and then divide by 3. Then subtract 2 from the result and tell me what your result is."*

This final number is always the age of the person involved.

Use the Algebra techniques shown above to see why this second trick never fails.

50 Card Shark

- Remove 26 cards from a standard deck.
- Split the 26 cards into two piles, one containing 10 cards, the other 16.
- Ask a friend to choose a card from the pack of 16 and then place it on the top of the deck of 10 cards.



- Place the remaining 15 cards on top of the deck of 11.

  To reveal the chosen card, deal the deck of 26 cards alternately into two rows face down.

- Once the twenty six cards have been dealt discard the thirteen cards in the left row.
- Collapse the right hand row, being careful to maintain the order of the cards.
- Repeat until there is only one card left. This will be the chosen card.

51 Hearing Things

Ask a friend to write down the following number

11 thousand, 11 hundred and 11

They will probably write down 11 111.

The number is in fact 12 111.

SOLUTIONS

1. Magic Squares

6	1	8
7	5	3
2	9	4

2	7	6
9	5	1
4	3	8

6	7	2
1	5	9
8	3	4

4	9	2
3	5	7
8	1	6

2	9	4
7	5	3
6	1	8

8	3	4
1	5	9
6	7	2

4	3	8
9	5	1
2	7	6

2. Making Magic Squares

4	9	2
3	5	7
8	1	6

8	3	4
1	5	9
6	7	2

6	1	8
7	5	3
2	9	4

3. Reflecting Magic Squares

4	9	2
3	5	7
8	1	6

2	9	4
7	5	3
6	1	8

8	3	4
1	5	9
6	7	2

4	3	8
9	5	1
2	7	6

6	1	8
7	5	3
2	9	4

8	1	6
3	5	7
4	9	2

The middle number in each magic square is 5. Multiplying the centre number by three gives the total for any row, column or diagonal.

4. Magic Square Investigation

When each number in a magic square is doubled the magic total for each row, column and diagonal is doubled.

Adding five to each number in a magic square still produces a magic square where the total for each row, column and diagonal equals three times the centre number.

Adding ten to each number in a magic square still produces a magic square where the total for each row, column and diagonal equals three times the centre number.

Subtracting a constant does not change the rule. Three times the centre number gives the total for each row, column and diagonal.

Multiplying by a constant produces a magic square where the total for each row , column and diagonal is 'n' x 3 x centre number (where n = constant).

5. Even-order Magic Squares

Thirty four.
Answers will vary. Reflecting a 4 x 4 magic square will produce a new magic square with a similar constant.

Adding or multiplying a constant will produce a magic square related to the first by the constant.

7. Magic Formula

The magic constant may be found by adding all eight original numbers that were substituted into the formula.

9. In the Long Run

In the long run the line will remain constant around the 0·5 mark.
The same thing will happen if tails are plotted.

10. Do or Die

The most likely roll to produce a six is the first roll.
A little bit of probability will explain why.

1st throw $^1/_6$

2nd throw $^5/_6 \times {}^1/_6$ or $^5/_{36}$. There are 5 chances out of six of not throwing a six on the first throw and then one chance out of six of throwing a six on the second throw.

The subsequent chances become even smaller
i.e. $^5/_6 \times {}^5/_6 \times {}^1/_6$ etc.

11. Baffling Birthdays

For a class of 'n' students the probability that two students will share the same birthday is given by

$$1 - \left(\frac{365}{365} \times \frac{364}{365} \times \frac{363}{365} \times \ldots \times \frac{365-n+1}{365}\right)$$

This means that for a group of 23 students the chance of two students sharing the same birthday is better than 1 in 2.

12. Fair Go

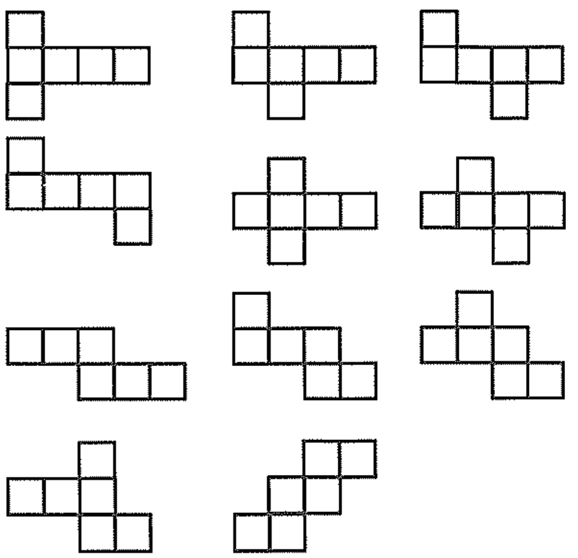
The chances of winning the game may be shown on a tree diagram.

H ($^1/_2$) win heads
T
H($^1/_4$) win heads
T ($^1/_4$) win tails

There is a 75% chance of winning with heads and only a 25% chance with tails. A point system that awards the tails player three points and the heads player one point should even things up.

16 Perplexing Polyhedron – The Hexahedron

There are eleven different nets that may be folded to form a cube.



17. Codes and Ciphers

CODED MESSAGES CAN BE VERY CONFUSING

18. Polybius Code

I LOVE MATHS

19. Codes and Ciphers II

ALL MATHEMATICS IS EASY

A	A	M	C	E
L	T	A	S	A
L	H	T	I	S
M	E	I	S	Y

20. Codes and Ciphers – ISBN

09588575 47

10(0) + 9(9) + 8(5) + 7(8) + 6(8) + 5(5) + 4(7) + 3(5) + 2(4) = 301
301 ÷ 11 = 27 remainder 4
11 – 4 = 7 — check digit is correct

0 9588575 9 x

as per above except ... 2(8) = 309
309 ÷ 11 = 28 remainder 1
11 – 1 = 10
x' represents 10

0 73164567 7

10(0) + 9(7) + 8(3) + 7(1) + 6(6) + 5(4) + 4(5) + 3(6) + 2(7) = 202
202 ÷ 11 = 18 remainder 4
11 – 4 = 7 check digit correct

0 64602382 9

10(0) + 9(6) + 8(4) + 7(6) + 6(0) + 5(2) + 4(3) + 3(8) + 2(2) = 178
178 ÷ 11 = 16 remainder 2
11 – 2 = 9 check digit correct

21. Codes and Ciphers

10 = 1010_2	$110\,010_2$	= 50
12 = 1100_2	$111\,111_2$	= 64
15 = 1111_2	$1\,000\,001_2$	= 65
19 = 10011_2	$1\,100\,100_2$	= 100
25 = 11001_2	$1\,011\,010_2$	= 106
28 = $11\,100_2$		
31 = $11\,111_2$		

22. Codes and Ciphers — Bar Codes

9312345 678907

0 + 8 + 6 + 4 + 2 + 3 = 23
23 x 3 = 69
9 + 1 + 3 + 5 + 7 + 9 = 34
34 + 69 = 103
103 + 7 = 110
(the nearest multiple of ten.)

therefore the check digit must be 7.

4006381 114615

1 + 4 + 1 + 8 + 6 + 0 = 20
20 x 3 = 60
4 + 0 + 3 + 1 + 1 + 6 = 15
60 + 15 = 75
75 + **5** (the nearest multiple of ten.) = 80

therefore the check digit must be 5.

9310353 080101

3 + 0 + 5 + 0 + 0 + 0 = 8
3 x 8 = 24
9 + 1 + 3 + 3 + 8 + 1 = 25
24 + 25 = 49
49 + **1** (the nearest multiple of ten.) = 50

therefore the check digit must be 1

9312650 901905

0 + 1 + 9 + 5 + 2 + 3 = 20
3 x 20 = 60
9 + 1 + 6 + 0 + 9 = 25
60 + 25 = 85
85 + 5 (the nearest multiple of ten.) = 90

the check digit must be 5.

23. Patterns in the Hundred Square

Multiplying the centre number of the square by 4 is a quick way of arriving at the total of the four numbers.
Algebraically this may be shown by labelling the first square 'a'.

a	a + 1	a + 2
a + 10	a + 11	a + 12
a + 20	a + 21	a + 22

$$a + (a + 2) + (a + 20) + (a + 22) = 4a + 44$$
$$4\,(a + 11) = 4a + 44$$

When using other odd sized squares the rule remains unchanged.

a	a + 1	a + 2	a + 3	a + 4
a + 10	a + 1	a + 12	a + 13	a + 14
a + 20	a + 21	a + 22	a + 23	a + 24
a + 30	a + 31	a + 32	a + 33	a + 34
a + 40	a + 41	a + 42	a +43	a + 44

$$a + (a + 4) + (a + 40) + (a + 44) = 4a + 88$$
$$4(a + 22) = 4a + 88$$

24. Patterns in the Hundred Square

Multiples of nine produce a diagonal pattern sloping from top right to bottom left, whereas multiples of eleven produce a diagonal pattern sloping from top left to bottom right.

Multiples of two appear in five columns (2, 4, 6, 8, 10).

Multiples of three appear in diagonals, three numbers apart.

Multiples of five appear in two columns (5, 10).

When multiples of nine are shaded, starting at six a diagonal pattern similar to straight multiples of nine is formed.

The digit sum for 6, 15, 24 etc. is always six.

Yes — when you constantly add nine, beginning with seven, the following series is formed 7, 16, 25, 34. The digit sum for all the numbers in this series is seven. The digit sum when beginning with eight and constantly adding nine is eight.

25. Patterns in the Hundred Square

When adding the values in opposite corners the totals are the same

A little algebra explains why.

a	a + 1	a + 2	a + 3	a + 4
a + 10	a + 11	a + 12	a + 13	a + 14
a + 20	a + 21	a + 22	a + 23	a + 24

$$a + (a + 24) = 2a + 24$$
$$(a + 4) + (a + 20) = 2a + 24$$

Multiplying the centre number by fifteen provides a quick way of finding the total of all the fifteen numbers in the rectangle.

The centre number in a 3 x 3 rectangle would need to be multiplied by nine to produce the total for all the numbers in the rectangle.

26. Patterns in Pascal's Triangle

The first diagonal is made up of ones.

The second diagonal is made up of counting numbers

The third diagonal is made up of triangular numbers.

The next four rows of the triangle are

1 5 10 10 5 1

1 6 15 20 15 6 1

1 7 21 35 35 21 7 1

1 8 28 56 70 56 28 8 1

A pattern is formed

1, 2, 4, 8, 16, 32, 64

The patterns may be described as powers of two.

27. Pascal's Triangle and Probability

p(2 heads) = $^1/_4$, p(1 head) = $^2/_4$, p(0 heads) = $^1/_4$,

p(3 heads) = $^1/_8$, p(2 heads) = $^3/_8$, (1) head) = $^3/_8$,

p(0 heads) = $^1/_8$. A pattern is formed. The numerators of the fractions match the third and fourth row of Pascal's Triangle. The total for the row gives the denominator. The sixth row of Pascal's Triangle can be used to work out the probability of getting 5 heads etc.

p(5 heads) = $^1/_{32}$, p(4 heads) = $^5/_{32}$, p(3 heads) = $^{10}/_{32}$,

p(2 heads) = $^{10}/_{32}$, p(1 head) = $^5/_{32}$, p(0 heads) = $^1/_{32}$.

$11^2 = 121$ (which is similar to the third row of Pascal's Triangle.

$11^3 = 1331$ (which is similar to the fourth row of Pascal's Triangle).

$11^4 = 14641$ (which is similar to the fifth row of Pascal's triangle).

The value of 11^5 is more difficult to predict because the sixth row of Pascal's Triangle is 1 5 10 10 5 1. $11^5 = 161051$. The connection may be seen if the tens are carried as in addition.

28. Patterns in Pascal's Triangle

A square number is produced every time.

29. Pascal's Stocking Pattern

The pattern works in all cases.

30. Figurate Numbers

Triangular numbers: 1, 3, 6, 10, 15, 21, 28, 36, 45, 55.

Adding two consecutive triangular numbers produces a square number e.g. 21 + 28 = 49

4 = 1 + 3, 9 = 3+ 6, 16 = 6 + 10, 25 = 10 + 15,

36 = 15 + 21, 49 = 21 + 28, 64 = 28 + 36, 81 = 36 + 45.

31. Fibonacci Numbers

Each new number in the sequence is made by adding the two previous numbers together. Consecutive numbers in the Fibonacci sequence produce a similar pattern.

32. Golden Rectangles

Comparing the length (10 cm) and the width (6 cm) of the remaining piece 10 ÷ 6, almost produces the golden ratio: 1 : 6.

33. Golden Fibonacci

The 8 x 5 rectangle conforms to the golden ratio. Adding an 8 x 8 square produces a 13 x 8 rectangle which also conforms to the golden ratio.

Adding a 13 x 13 square produces a rectangle that is 21 x 13 which conforms to the golden ratio.

Adding a 21 x 21 square produces a 34 x 21 rectangle which also conforms to the golden ratio.

34. Calendar Capers

Following the procedure always brings you back to the starting date in the block of four.

35. Calendar Corners

The numbers in opposite corners always add to the same total. A little algebra explains why this happens. Let "a" represent the first date in a block of nine.

a	a+ 1	a + 2
a + 7	a + 8	a + 9
a + 14	a + 15	a+ 16

$a + (a + 16) = 2a + 16$

$(a + 2) + (a + 14) = 2a + 16$

36. Calendar Perimeter

The answer is always eight.

37. Calendar Count

Each row increases by 21 (i.e. 3 weeks or 3 x 7).
Each column increases by 3 (i.e. 3 days).

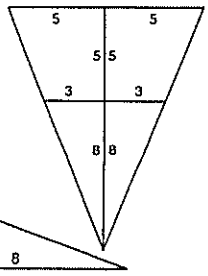
38. Calendar Cross

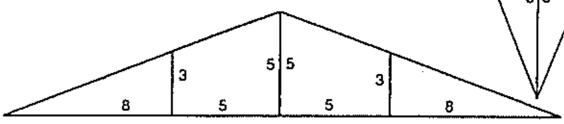
The values along each diagonal are the same. If you multiply the middle number by 3 you can find the total for each diagonal.

39. Missing Areas

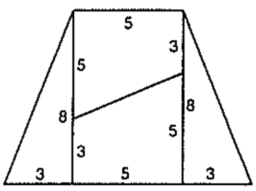
Area rectangle = 65 square units

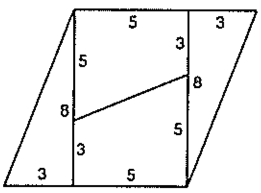
The area of both triangles is 65 square units





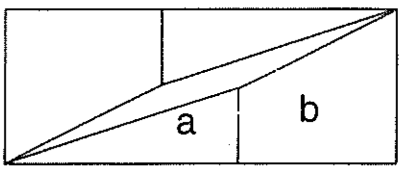
Area Trapezium = average of parallel sides x perpendicular height
$= (5 + 11) \div 2 \times 8$
= 64 square units





Area parallelogram = base x perpendicular height
= 8 x 8
= 64 square units

The reason for this apparent contradiction lies along the diagonal



The pieces don't really fit together and so a small parallelogram with an area of one square unit is formed. You may also like to use trigonometry to work out the angle measurements where the triangle and the trapezium join to form a straight line. You will find the two angles 'a and b' add to slightly more than 180 degrees

40. Prime Time

A property of prime numbers is highlighted by this trick. To illustrate mark some prime numbers on a six column grid.

1	2	3	~~4~~	5	~~6~~
7	~~8~~	~~9~~	~~10~~	11	~~12~~
13	~~14~~	~~15~~	~~16~~	17	~~18~~
19	~~20~~	~~21~~	~~22~~	23	~~24~~
~~25~~	~~26~~	~~27~~	~~28~~	29	~~30~~
31	~~32~~	~~33~~	~~34~~	~~35~~	~~36~~
37	~~38~~	~~39~~	~~40~~	41	~~42~~
43	~~44~~	~~45~~	~~46~~	47	~~48~~
~~49~~	~~50~~	~~51~~	~~52~~	53	~~54~~

Note that the primes greater than three end up in the five column or one other column. we can generalise that any prime greater than three is of the form $6n \pm 1$

- Squaring the number gives $36n^2 \pm 12n + 1$
- Adding 15 gives $36n^2 \pm 12n + 16$
- Dividing by twelve will therefore always leave a remainder of 4.

Note altering the number added will change the remainder.

41. Multiple Madness

15873	x	7			=	111 111
15873	x	7	x	5	=	555 555
15873	x	7	x	n	=	nnn nnn

42. Multiple Madness II

8547	x	13			=	111 111
8547	x	13	x	3	=	333 333
8547	x	13	x	n	=	nnn nnn

Students can create their own 'tricks' of this type using similar principles eg

111 111	÷	3	=	37 037
111 111	÷	11	=	10 101
111 111	÷	37	=	3003 etc

43. Number Novelties

Let abcd represent the digits of the starting number. The four digit number would therefore be represented by

$1000a + 100b + 10c + d$

Interchanging the digits and subtracting the smaller number from the larger produces

$$1000a + 100b + 10c + d$$
$$-(1000d + 100b + 10c + a)$$
$$= 999a - 999d$$
$$= 999(a - d)$$

If a = d ie the first and last numbers are the same then the result will be zero. If the difference between the first and last digits is one the answer will always be 1998 (ie 999 + 999). The result for all other values will be 10 989.

44. 9 Guzinta

A little algebra helps to explain this. If a and b represent the digits we get $10a + b - (a + b) = 9a$.

Therefore the result is always divisible by 9.

45. 99 Guzinta

If a, b and c represent the three digits we get:

$$\begin{aligned}(100a - 10b + c) - (100c + 10b + a) &= 99a - 99b \\ &= 99(a - c)\end{aligned}$$

Therefore the result is alwayus divisible by ninety-nine.

46. Multiplication Shortcuts

$(a + b)^2 = a^2 + 2ab + b^2$

eg $45^2 = (40 + 5)^2$
$= 2025$

47. Multiplication Shortcuts II

$(a + b)^2 = a^2 + 2ab + b^2$

$56^2 = (50 + 6)^2$
$= 3136$

48. Mind Reader

Let '*n*' denote the chosen digit. The steps produce $(10n + n) \times 9 \times 11 = 1089n$. This number is always divisible by nine and eleven. This means that the sum of the digits that make up the number will equal nine or a multiple of nine. $89 \times n$ always produces a number where the tens digit is one less than the units digit eg $4 \times 89 = 356$. The units digit and the hundreds digit will always add to nine. The zero in 1089 ensures that no digits are carried over into the thousands place. The thousands digit will be the same as the single digit multiplier (n), because $1 \times n = n$. The thousands digit and the tens digit add to make nine.

49. Hearing Things

A person is often tricked into writing down the wrong number because he/she focusses on the place value signals.

50. Card Shark

Drawing a table which locates the required card in the 16th position on the first sorting will help to show that successive sortings based on removing the left hand column of cards eventually leaves only one card remaining in the right hand column. The first deal produces 13 cards in each row. The left row is discarded. The second deal produces 7 cards in the left row and 6 in the right. The third deal produces three cards in each row and so on until a single card is left.

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