

Paul Swan, Linda Marshall.

Geometry Review Guide

The intention of this **Review Guide** is to assist you to find your strengths and weaknesses in Geometry. The booklet is divided into aspects of Geometry: **2D Shape**; **3D Shape**; **Similarity and Scale**; **Symmetry**; **Co-ordinates and Transformations**. Test yourself on the Geometry Check before you read through the rest of the guide. Once you have checked your answers, work through the sections where you need assistance.

* For further information, see Bana, J., Marshall, L., and Swan, P. (2005). Maths Terms and Tables. Perth: R.I.C. Publications.

Paul Swan, Linda Marshall.

I

Test yourself on the Geometry Check before you read through the rest of the guide. Once you have checked your answers, work through the sections where you need assistance.

Снеск

2.

1.

- The diagrams show
 - (a) triangular prisms
 - (b) rectangular prisms
 - (c) triangular pyramids
 - (d) rectangular pyramids
 - (e) none of these



Swan and Marshall

- The diagram shows a solid which has
 - (a) 10 vertices and 15 edges
 - (b) 9 vertices and 12 edges
 - (c) 15 vertices and 7 edges
 - (d) 15 vertices and 10 edges
 - (e) none of these



- (a) 4 horizontal and 1 vertical face
- (b) 1 horizontal and 2 vertical faces
- (c) 1 horizontal and 4 vertical faces
- (d) 2 horizontal and 1 vertical face
- (e) none of these



Х

4. Which shape would be formed if the triangle drawn here spun very fast around the axis through XY?



is

Here is a drawing of a pentagonal prism.

Which outline below could be cut out and folded along the dotted lines to make the best model of this pentagonal prism?



K

G

35

F

Ν





J

L

45

Η

- 6 Which shapes drawn here are parallelograms:
 - (a) all the shapes
 - all except H and I (b)
 - F and G only (c)
 - (d) F only
 - none of these answers (e)
 - The size of angle M in triangle LMN is
 - (a) 80°
 - (b) 90°
 - 100° (c)
 - (d) 110°
 - none of these (e)



190°

- 8. What is the size of angle Q?
 - (a) 70° (b) 90°
 - (c) 100°
 - (d) 110°
 - none of these (e)

3D Objects

7

9. To move the shape from position G to position H, slide it

- (a) down 2, right 5
- (b) left 5, up 2
- (c) down 1, right 1
- (d) down 2, right 1
- (e) none of these



10. Which picture shows the correct position of the mirror image for the object O?



Geometry Check

V

11. The shaded part of each picture shows a cardboard shape pinned at point P. The card is given a1/4 turn clockwise. Which dotted outline shows the new position of the card?



12. Which picture could be cut and folded so that both halves match?



(e) none of these



Symmetry

Co-ordinates

Geometry Check





- (a) Start at (2, 3) and go to (5, 3) then finish at (2, 6)
- (b) Start at (2, 3) and go to (3, 5) then finish at (6, 2)
- (c) Start at (3, 2) and go to (5, 3) then finish at (2, 6)
- (d) Start at (6, 2) and go to (3, 5) then finish at (2, 2)
- (e) none of these



15. Name the point at (4, 2) and give the co-ordinates of point D.

- (a) B, (3, 1)
- (b) C, (1, 3)
- (c) B, (1, 3)
- (d) C, (3, 1)
- (e) none of these



16. How many of the shape N would be needed to make shape M?



End of Geometry Check...

Turn to the next page to check your answers >>>

>>> Answers to Geometry Check

d 1. 2. a 3. C 4. b 5. a 6. C 7. *C* 8. C 9. a 10. d 11. a 12. a 13. d 14. b 15. a 16. a

How did you do?

Use the section indicators in the left column of the Geometry check to locate the sections you would like to improve on.

SECTION 1. TWO DIMENSIONAL SHAPES



TWO DIMENSIONAL SHAPES

Two dimensional shapes that we will investigate in this section are called **polygons**. 'Poly' means **'many'** and 'gon' means **'side'**, e.g. many sided figures. Two dimensional shapes have length and width, e.g. a rectangle or square has length and width, a triangle has base and height.

Poly: "many" Gon: "sides"



The main shapes that you should recognise are:

	· • · · · · · · · · · · · · · · · · · ·
3 sides	triangles
4 sides	quadrilaterals
5 sides	pentagons
6 sides	hexagons
8 sides	octagons
10 sides	decagons

Regular polygons are those that have all sides congruent (i.e. of equal length) and all angles congruent (i.e. of equal size).

See p. 87 of Maths Terms and Tables.



Triangles are three sided polygons.





Scalene

Isosceles (two congruent sides)



Equilateral (all sides and angles congruent) (this is a regular polygon)

Congruent shapes have the same size and shape.



Quadrilaterals are four sided polygons.



Quadrilateral - any four sided figure





This is a regular quadrilateral.

Rectangle - opposite sides congruent each angle is 90°

Parallelogram - opposite sides congruent and parallel opposite angles congruent



Rombus - all sides congruent opposite sides parallel opposite angles congruent



Trapezium- only one pair of parallel sides *See p. 92 of Maths Terms and Tables.



Pentagons are five sided polygons.



Non-regular



Regular - all sides congruent all angles congruent (108°)



Hexagons are six sided polygons.



Non-regular

Regular- all sides congruent all angles congruent (120°)



Octagons are eight sided polygons.





Regular- all sides congruent all angles congruent (135°)



Decagons are ten sided polygons.







Regular- all sides congruent all angles congruent (144°)



• Intersecting lines are those that cross each other at any point • Bisecting lines cut each other in half

OTF

J

Check

int

С

F

Ι



4 Classify the following shapes according to the number of sides;





5

True or false:

i) Al	l trapeziums	are paralle	lograms
-------	--------------	-------------	---------

- ii) All rectangles are parallelograms.
- iii) All parallelograms are rhombuses.
- iv) All squares are rhombuses.
- v) All trapeziums are quadrilaterals.



- *iii)* False
- iv) True
- v) True

SECTION 2. THREE DIMENSIONAL OBJECTS



THREE DIMENSIONAL OBJECTS

Three dimensional shapes that we will investigate in this section are called **polyhedra**.

Poly means 'many' and hedra means 'face\$'.





3D objects are those that have length, width, & height.

These are polyhedra. Notice that all the **faces** are **polygons**.

The main 3-dimensional shapes that you should recognise are *prisms* and *pyramids*.





Prisms are shapes with two opposite ends being congruent polygons and all other faces being rectangles. Notice that the two triangles are the same shape and size (.i.e. congruent) and the other three sides are rectangles.





If we decide to unfold the triangular prism we would obtain the following shape:



This is called a **NET** and it is a **two dimensional representation** of a **three dimensional object**.

Here is another example of a **NET** created from a **hexagonal prism**





Name each shape below and determine the number of vertices, faces and edges.

The information for the triangular prism has been completed for you.

Name	No. of Vertices	No. of Faces	No. of Edges	
A. Triangular Prism	6	5	9	
В.				
С.				
D.				
E.				

Challenge 1: Examine the numbers in the columns and see if you can discover a relationship between them.





A pyramid consists of a base (a polygon) and triangular faces which meet at a point called an apex.

Pyramids are named by their bases Apex square pyramid triangular pyramid hexagonal pyramid

If we decided to unfold the square pyramid we would have the following net:



An important aspect of your pyramid is the number of *vertices* (corners), *edges* and *faces*.

For example: the square pyramid has:

5 vertices or corners

5 faces

8 edges





vertex



Name each shape below and determine the number of vertices, faces & edges.

The information for the square pyramid has been completed for you.

	1	1		_
Name	No. of Vertices	No. of Faces	No. of Edges	
A. Square Pyramid	5	5	8	
В.				
С.				
D.				
E.				

Challenge 2: If you discovered the relationship between the numbers for the prisms, can it be applied here?











* Euler's Law indicates the relationship between Vertices, Faces and Edges V + F = E + 2



Besides pyramids and prisms you should be able to recognise *cones, cylinders* and *spheres*.

CONES

Cones are like pyramids.

They have a circular base.

THEY ARE EASY TO REMEMBER - JUST THINK OF ICE-CREAM CONES

CYLINDERS

Cylinders are like prisms.

They have two congruent circular ends.



A COOL DRINK CAN IS AN EXAMPLE OF A CYLINDER

SPHERES

Spheres aren't like prisms and pyramids.



A BALL IS A GOOD EXAMPLE OF A SPHERE.



Pg. 12

Nai	me	No. of Vertices	No. of Faces	No. of Edges
A. Tria	angular Prism	6	5	9
B.	Square Prism (cube)	8	6	12
C.	Rectangular Prism	8	6	12
D.	Hexagonal Prism	12	8	18
E.	Pentagonal Prism	10	7	15
Cha	allenge 1: V +	F = E + 2 or $V + F$ -	E = 2	

Pg. 14

Nan	ne	No. of Vertices	No. of Faces	No. of Edges				
A. Squ	are Pyramid	5	5	8				
В.	Triangular Pyramid	4	4	6				
C.	Hexagonal Pyramid	7	7	12				
D.	Rectangular Pyramid	5	5	8				
E.	Pentagonal Pyramid	6	6	10				
Cha	llenge 1: Yes,	$\mathbf{V} + \mathbf{F} = \mathbf{E} + 2$						



CONGRUENCE, SIMILARITY & SCALE

CONGRUENCE, SIMILARITY, AND SCALE



Two shapes are said to be **congruent** if they are the *same shape and size*. The following pairs of shapes are congruent:



You can test for congruence by measuring or more simply in two dimensions by super-imposing one shape on top of the other.

IMILARITY

Two shapes are said to be **similar** if they have the *same shape*. The same shape means that the corresponding angles are congruent and the corresponding sides are in the same proportions. More simply one is a larger scale model of the other, e.g.







Notice that all squares and *regular* hexagons would be similar.

Not all triangles are similar, e.g.



- the corresponding angles aren't congruent and the corresponding sides are not proportional.

Congruence

Similarity

Scale



However, some triangles may be similar, e.g.







Congruence

20

Scale



A practical and realistic way in which you can be introduced to the idea of similarity is by scaling. Most people are familiar with the world of scale in two and three dimensions through model making, maps and photographs, e.g.



Rectangle B is a **double scale** model of rectangle A. Notice that the lengths of sides of rectangle B are twice as long as the sides of rectangle A. **How many rectangles the size of A would fit into B? Therefore what is the increase in area?**

Two ways of constructing scale diagrams, in two dimensions are: 1. use of grids 2. use of centre of enlargement



21

Scale



Complete this table 2.

Scaling Factor	Length of Side	Area
2	doubles (2 x)	quadruples (4 x)
3		
	4 x	
	5 x	
1/2		
1/3		

Congruence

Similarity

22



2) Use of centre of enlargement

From a point inside or outside the figure (called the centre of enlargement) we can draw a scaled model of the figure. E.g.

(i) Inside the figure

Double scale model of WXYZ



(ii) Outside the figure

Double scale model of PQR
Place Centre C anywhere outside the figure.
Join C to each vertex and double the distance to obtain P'Q'R'.

23



Scale

25





		/
Scaling Factor	Length of Side	Area
2	doubles (2 x)	quadruples (4 x)
3	trebles (3 x)	nine times (9 x)
4	4x	sixteen times (16 x)
5	5x	twenty five times (25 x)
1/2	halves (1/2)	one quarter (1/4 x)
1/3	<i>third</i> (1/3)	one ninth (1/9 x)







SYMMETRY

Many shapes in the environment are symmetrical. Consider this picture of a butterfly:



INE OF SYMMETRY OR REFLECTION / BILATERAL SYMMETRY

Shapes that have reflection symmetry are those where one half of the shape is a mirror reflection of the other half, e.g.



The reflection line is called the line or axis of symmetry.

Some shapes have more than one line of symmetry, e.g.





MIRA

This is a small



To test for reflection symmetry you can:

a) Use a mirror or *mira* to see if one half can be reflected onto the other half.



If this is possible then the fold line is the line of symmetry.



OTATIONAL SYMMETRY OR TURN SYMMETRY

Fans and windmills are examples of objects with rotational symmetry. A shape can be tested for rotational symmetry by tracing the shape, and rotating the tracing around the original shape. If the shape matches other than in the original position then the shape has rotational symmetry.



the point about which the shape is rotated is called the centre of rotation

The rectangle has an order of rotational symmetry of 2, i.e. if you turn the rectangle through 180° it will match the original shape.



NOTE:

All shapes will match if you turn the tracing through 360°. If this is the only place where they match, then these shapes don't have rotational or turn symmetry.

Order of Rotation

The number of times a figure appears to retain its original orientation during one complete rotation about a fixed point.



2 Decide which letters have:

a) reflectional symmetry only

b) rotational symmetry only

c) both reflectional and rotational symmetry

d) no symmetry

C F R Η F G Κ LM S R VWXYZ

3 Would the font style affect your results?

YMMETRY & 3D OBJECTS

Three dimensional shapes may also have reflection symmetry and/or rotational symmetry, eg.



If we cut here, we have reflection symmetry. (Note: planes of symmetry)



If we turn this shape about this line 180° we have rotational symmetry.



- 1. List some three dimensional shapes or objects in the environment with:
 - a) reflection symmetry
 - b) rotational symmetry
- 2. How does symmetry differ for three dimensional objects in comparison to two dimensional shapes?

Symmetry-solutions Check Point

Pg, 30









Pg, 31

- 1a) rotational symmetry of 90°
- 1b) rotational symmetry of 60°
- 1c) rotational symmetry of 45°
- 1d) rotational symmetry of 180°
- 1e) no rotational symmetry
- 1f) rotational symmetry of 180°

Pg, 32

- 2a) reflectional symmetry only
- 2b) rotational symmetry only
- 2c) reflectional & rotational symmetry
- 2d) no symmetry

- ABCDEKMTUVWY NSZ HIOX BFGJLPQR
- 3) yes- the type of font will make a difference, eg. B , ${\boldsymbol{B}}$

Pg, 33

- 1a) Depends on the shapes, eg. doors, desks, cupboards.
- 1b) Fans, windmills, pipes

2)

	2D	3D
Reflectional Symmetry	line of symmetry	plane of symmetry
Rotational Symmetry	centre of rotation	line or axis of rotation





CO-ORDINATES

Co-ordinates are a way of assigning numbers (or letters) to points on a plane. Co-ordinates are met frequently in real life situations such as, in maps and street directories.

In locating the point A(2,3) by convention, the first number is the horizontal co-ordinate (x co-ordinate) and the second number is the vertical co-ordinate (y co-ordinate).



- 1 On a sheet of grid paper, plot the following sets of points joining consecutive points as you go.
 - a) (1, 7), (-4,3), (-5, -2), (0,2), (1, 7)
 - b) (-5, -2), (0,-6), (5, -2), (6,3), (1, 7), (0,2), (5, -2)
 - c) (-4,3), (1,-1), (6, 3)
 - d) (1,-1), (0, -6)

What is the resultant shape?



2

On a sheet of grid paper, plot the points:

A(-3,5), B(3,7), C(7,-3), D(-5,-1)

a) What is the resultant shape?

b) List the co-ordinates of the mid points of AB, BC, DC, and AD. *c)* What shape is formed by joining these mid points?

3 Describe the directions you would give using co-ordinates, if you travelled from A to B along the path shown.



4 On a sheet of grid paper, plot the points: A(2,6), B(4, -4), C(-6,-2)

What are the co-ordinates of:

- a) *M, the mid-point of AB?*
- b) N, the mid-point of AC?
- c) What type of figure is NMBC?
- d) How does the length of MN compare with BC?



Pg. 42

1) A cube

Pg. 43

- 2a) a quadrilateral
- 2b) (0,6), (5,2), (1,-2), (-4,2)
- 2c) parallelogram

3) Start at (1,5) then go to (2,1), then go to (5,2) and then finish at (4,3).

- 4a) M (3,1)
- 4b) N (-2,2)
- 4c) Trapezium
- 4d) a half



TRANSFORMATIONS

TRANSFORMATIONS

A transformation is the movement of a shape from one position to another. The movement of shapes can be derived from three basic moves:

i) A *reflection* or *flip* is where a shape is reflected (or flipped) about a line, called the line of reflection, to a new position.



The shape is just as far in front of the line of reflection as behind it. A good example of a reflection or flip is when you look in the mirror. So another way of describing a reflection is by calling it a mirror image.

*Note that the line if intersection may intersect the shape.





ii) A *translation* or *slide* is where a shape is translated or slid, to a new position.



The shape is not 'turned' or 'twisted' in any way and that the slide can be in any direction.

An example of a translation or slide would be an escalator in a shopping centre. People are translated from one level to another.



Old position

New Position

Another example would be children on a slide in a playground.



iii) A shape may be rotated or turned.

For example:



*Note that the centre of rotation may be inside the shape.



Good examples of rotations or turns in the environment would be the fan blades of windmills. fans, propellers, etc.

Translations, rotations and reflections involve changes in location only and do not involve changes in size and shape. The shape is not changed, therefore the two shapes are congruent.

42



Mira

a)

c)









270° Anti-clockwise

4. Classify the following movements as reflections, rotations or translations



5. In each of these sketches, the shaded flag has been moved to a new position. Decide in each case whether the transformation is a reflection, a rotation, a translation, or some combination of these. Use a mira, mirror, or tracing paper if you wish.



Check Point

6. Show how the shape in each case can be moved from position 1 to position 2.



- 7. In summary, what information must be given for a shape to under go:
 - a) a reflection

RANSFORMATIONS

- b) a translation
- c) a rotation





48

					 	<u> </u>			

Т																																	
ŧ	_	_																															
l																																	
t																																	
ł	\rightarrow	_				<u> </u>	<u> </u>			\vdash														_						\vdash			
l																																	
I																																	
t	\neg																		_														
ł	\rightarrow	_																						_								$ \rightarrow $	
l																																	
Î																																	
ł	\rightarrow	_					<u> </u>			\vdash									_					_						\vdash	_		
l																																	
l																																	
t	\neg									\vdash	_					_			_		_	_			_			_					
ł	\rightarrow	_																						_								<u> </u>	
T																																	
ł	\rightarrow		\vdash				-			\vdash					\vdash				_					-						\vdash	-		
ļ	$ \rightarrow$	_																						_								$ \longrightarrow $	
I																																	
t	\neg																		_														
ł	\dashv	_	\vdash	<u> </u>			<u> </u>		\vdash	\vdash			\square	\square	\vdash		\square		_					_			\square		\square	\vdash	\neg	$ \rightarrow $	
ļ																																	
ľ	ſ]]]]]]]]]]]]]	I]]]]]]		Ī	
t	\dashv		\vdash							\vdash					\square															\vdash	\neg		
ł	\rightarrow	_	$ \square$	<u> </u>		<u> </u>	<u> </u>			\vdash					$ \square$									_						\vdash	$ \rightarrow$		
l																																	
T																																	
t	\neg									\vdash									_			_		-	_					\vdash			
ł	\rightarrow	_																						_								<u> </u>	
l																																	
T																																	
t	\rightarrow	_								\vdash	_					_			_		_	_		_	_			_		\vdash			_
ļ	$ \rightarrow$	_																						_									
l																																	
Î																																	
ł	\rightarrow	_																	_					_									
l																																	
l																																	
t																			_														
ł	\rightarrow	_			<u> </u>	<u> </u>	<u> </u>	<u> </u>		\vdash														_						\vdash		<u> </u>	
l																																	
ſ	T																																
t	\dashv		\vdash							\vdash					\vdash				_	\square										\vdash	\neg	\neg	_
ł	\rightarrow					<u> </u>	<u> </u>			\square														_						\square			
l																																	
T																																	
t	+		\vdash							\vdash			\square		\vdash				_								\square			\vdash	\neg	-+	_
ļ	$ \downarrow$																																
t																																	
ł	\dashv	_	\vdash	<u> </u>		-	-		\vdash	\vdash					\vdash				_					_						\vdash	-	$ \rightarrow$	
ļ																																	
I																																	
t	\neg																		_														_
ł	\rightarrow	_	\vdash			-	<u> </u>		\vdash	\vdash					\vdash									_						\vdash		$ \rightarrow$	
l																																	
ſ	T																						T									T	
t	\dashv		\vdash	-						\vdash					\vdash				_	\square					_					\vdash	\neg	\neg	_
ł	\rightarrow	_				<u> </u>				\square														_						\square			
J						L																											
T																																	
t	\dashv		\vdash			-				\vdash					\vdash												\square			\vdash	-	\neg	
ļ																																	
I																																	
4.																																	





► F

ANGLES

Measuring Angles

An angle is made up of two arms (or rays) and a corner (or vertex).



Notice how the angles are named

∠ABC is a 'sharp' point

∠DEF is a 'blunt' point

Angles are given particular names associated with their size.

Sharp angles between 0-90° are called acute angles.



Blunt angles between 90° - 180° are called obtuse angles.

The most common angle we use is the right angle or 90° angle. We see it where a floor meets the ceiling, in corners of rooms, in squares and rectangles, etc. The angles below are all right angles.



53

180° angles are straight angles.

Angles between 180° - 360° are called reflex angles.

An angle can be measured using a protractor. Semi-circular protractors may be found in many classes. However it makes more sense to use a full-circle protractor, when measuring reflex angles.





1. Write the correct label for each of these angles:





2. Estimate the size of these angles and write the type of angle next to each of them.



3. Now measure the actual size of each angle.

4. Construct each of these angles.a) 90°b)

b) 82°

c) 125°

d) 173°

55



When two straight lines cross each other, there are pairs of angles formed. The opposite angles are always the same size. They are called vertically opposite angles. The 'vertically' refers to the vertex or the point where the lines cross each other.



Angle $a = Angle b (\angle a = \angle b)$ Angle $c = Angle d (\angle c = \angle d)$

5. Write the correct angle size for both of these.





Adjacent angles are two angles in the same plane with a common side and a common vertex (point). Angles JKM and MKL are adjacent angles.



Complementary angles are two angles that together make a right angle (90°). Angles DEF and FEG are complementary.



Supplementary angles are two angles that together make 180°; e.g. two right angles are supplementary; angles ABC and CBD are supplementary.





6. Name the size of these angles.



7. What size are angles x, y and z?





Check Point 1

- 1. Answers as per angles in pics.
- 2. Compare estimates with actual measures (below).
- 3. a) 37° b) 242° c) 27° d) 89°
- e) 90° f) 119°
- 4. Correct angle sizes.
- 5. 67° 110°
- 6. a) 40° b) 37° c) 156°
- 7. $\angle x = 52^{\circ}; \angle y = 128^{\circ}; \angle z = 52^{\circ}$