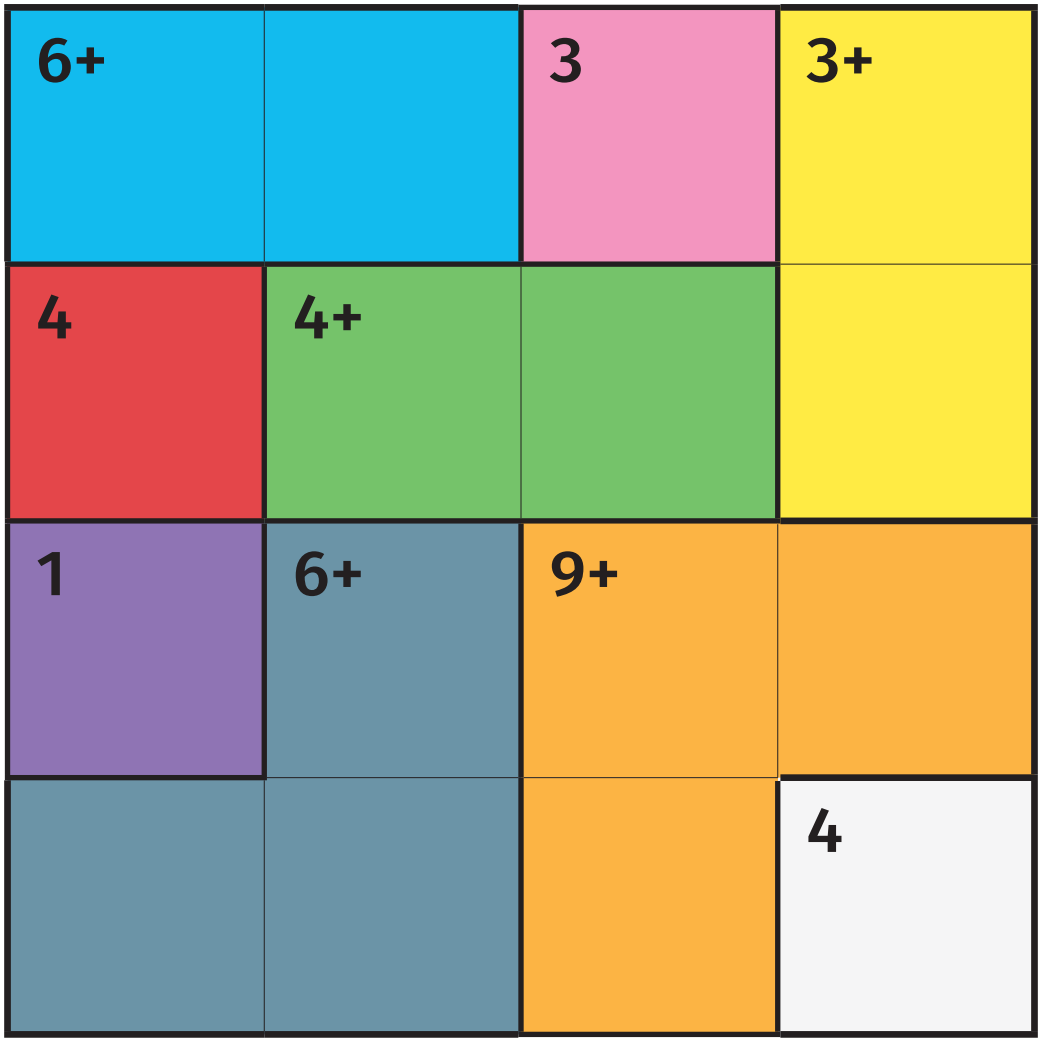
Building Fluency and Reasoning Using KenKen® Puzzles



David Dunstan & Paul Swan

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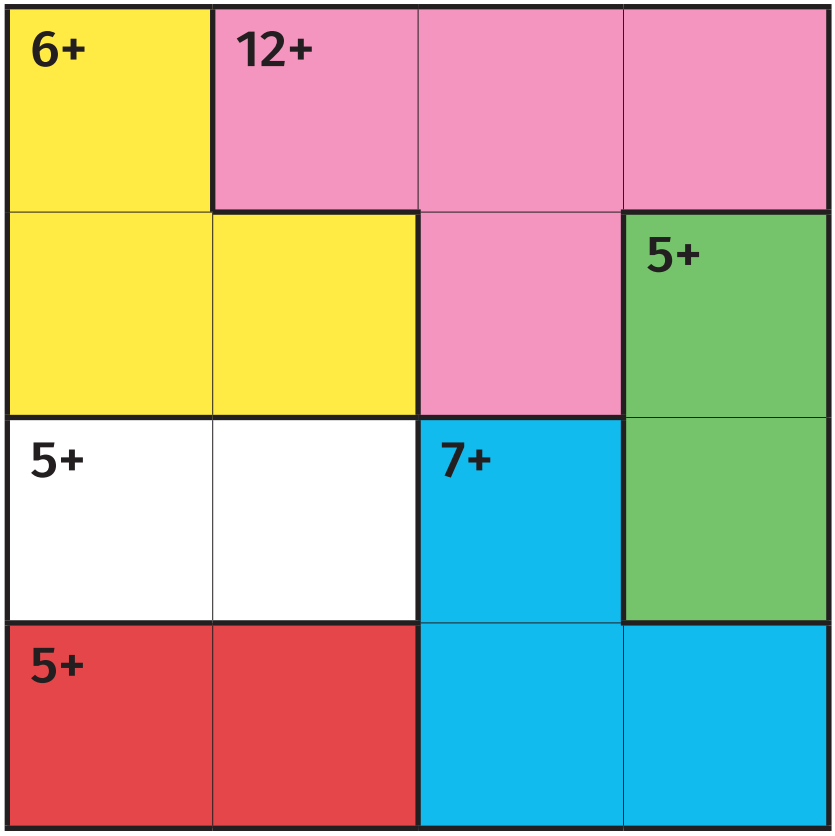
Acknowledgements

Reference: Using KenKen to Build Reasoning Skills

Reiter, H.B, Thornton, J. and Vennebush, P.

December 2013/January 2014, *Mathematics Teacher* 107, 5., pp.341-347



Background

In 2004, Tetsuya Miyamoto created and developed KenKen for use in the classroom to foster the skills of logic, guess and check and persistence.

KenKen puzzles vary according to:

- The size of the grid;
- The level of difficulty;
- The arithmetic operations;
- The number of "freebies" (single cell cages);

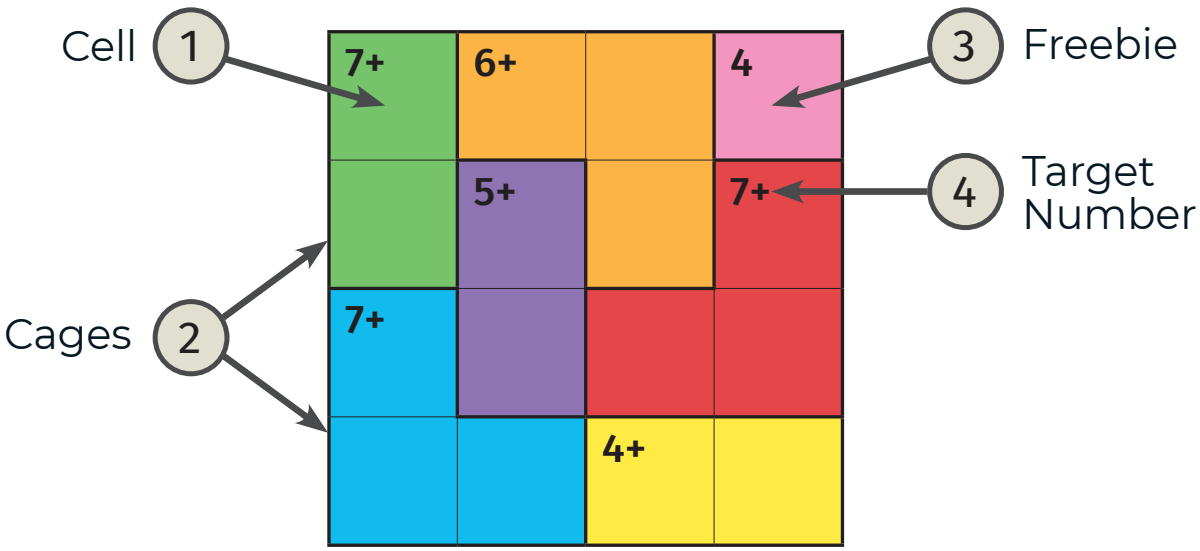
Each KenKen has only one solution.

How to Play

- The numbers used are determined by the size of the KenKen puzzle. For example, in a 6 x 6 puzzle, the numbers 1 to 6 inclusive, are used.
- Numbers cannot be repeated in any row or column.
- Fill each cage as determined by the Target Number and its accompanying arithmetic operation (if it has one).

Terminology

(1) Cell	A single square in the cage or grid.
(2) Cages	Heavily outlined shape showing a group of cells.
(3) Freebie	A single cell stating a particular candidate (number).
(4) Target Number / Operation	The top left hand corner of each cage has a Target Number and an arithmetic operation symbol (if it is not a Freebie).
(5) Row	A horizontal line of cells.
(6) Column	A vertical line of cells.



Playing Tips

- Firstly, complete any “Freebie” cages.
- Look for cages where there is only one possible solution.
- Subtraction is related to the difference between the numbers.
- The order of the digits for subtraction and division does not matter.
- Digits can be repeated in a *cage*, as long as there are no row or column repeats.

Educational Applications

KenKen offers a whole school approach for developing the proficiencies of

- Fluency;
- Problem Solving; and
- Reasoning.

The puzzles can be solved independently or in pairs. The benefits of working in pairs are:

- increased student mathematical dialogue;
- taking turns to verbalise reasoning;
- sharing ideas and strategies and a positive disposition through socialising.

Differentiation

Grid Sizes: Grid sizes generally range from 3 x 3 to 9 x 9.

Operations: There are generally four different puzzle types.

- Addition (+)
- Addition and Subtraction (+ -)
- Multiplication (x)
- All Operations (+ - x ÷)

Difficulty Level:

There can be five levels for each grid size: Easiest, Easy, Medium, Hard and Expert.

More Information and Puzzles

www.kenken.com

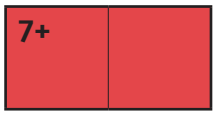
Students can be taught to reason by making various strategies explicit.

Row and Column Totals:

- 3 x 3 grid 1 + 2 + 3 = **6**;
- 4 x 4 grid 1 + 2 + 3 + 4 = **10**, ...
- 9 x 9 grid = **45**.

Unique candidate sets: (In a 4 x 4 grid)

For a two cell cage with a target number of 7+, only the digits 3 and 4 can be used.



Candidates:

{3, 4} and {4, 3}

Two possible candidate sets: (In a 4 x 4 grid)

For a two cell cage with a target number of 5+, there are two partition pairs for 5.

Candidates:

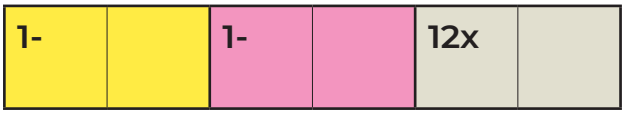
{1, 4} and {4, 1}

{2, 3} and {3, 2}

Parity – the cage value is odd or even.

Example 1: In a two cell cage where the target number is 4 and the operation is subtraction (difference), both digits are even or both are odd.

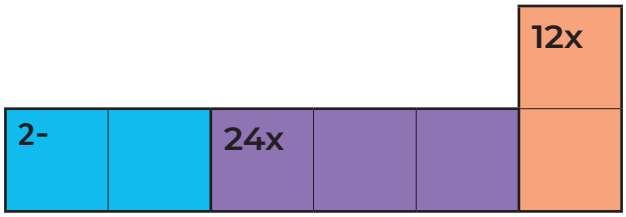
Example 2: For a 6 x 6 grid the digits used are 1, 2, 3, 4, 5 and 6; that is 3 odd and 3 even digits.



Cage is ODD (1 even and 1 odd digit); As there are two (1-) cages, 2 even and 2 odd digits will be used.

In the **(12x)** cage, the possibilities are {3,4} and {2,6}. This cage must also be ODD, thus {3,4} are the factors.

Example 3: 6 x 6 grid.



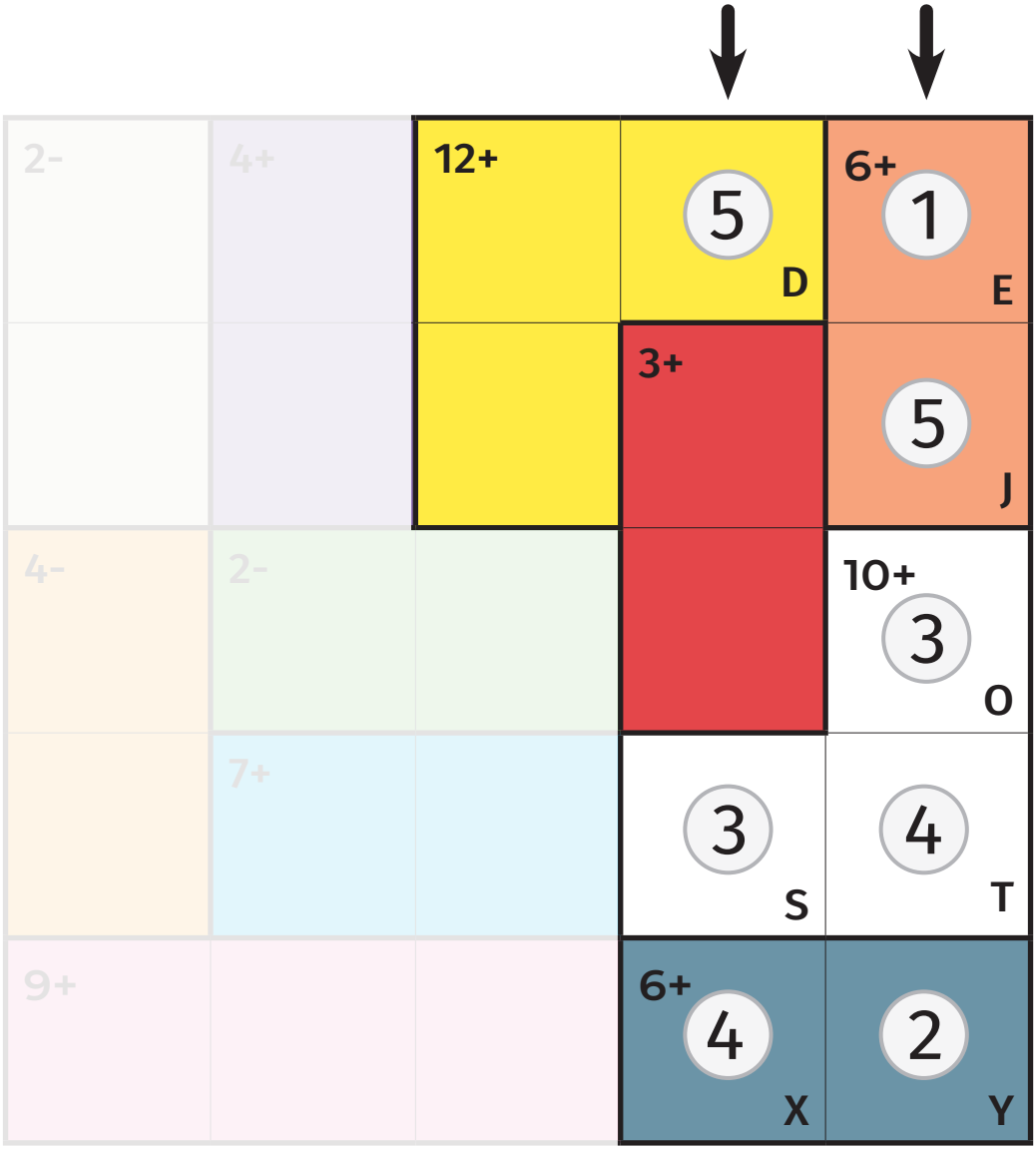
For the **difference of 2 cage (2-)**, {1,3} or {3,5}; there must be a 3.

For the **(24x) cage**, {1,4,6} or {2,3,4} triples are possible; there must be a 4.

Because of these 2 cages, **(24x)** must be 1,4,6, so **(2-)** must be 3,5.

Solving a 5 x 5 Sum and Difference KenKen: One Approach

Solving and Reasoning Steps - Part 1



The sum of every row and column is 15 (1+2+3+4+5).

The total for the last two columns on the right is 30.

- Therefore cell D **(yellow)** is 5 [30 - (6 + 3 + 10 + 6)]

In column 4, cell X **(grey)** is 4 (as it cannot be 3, 2 or 1).

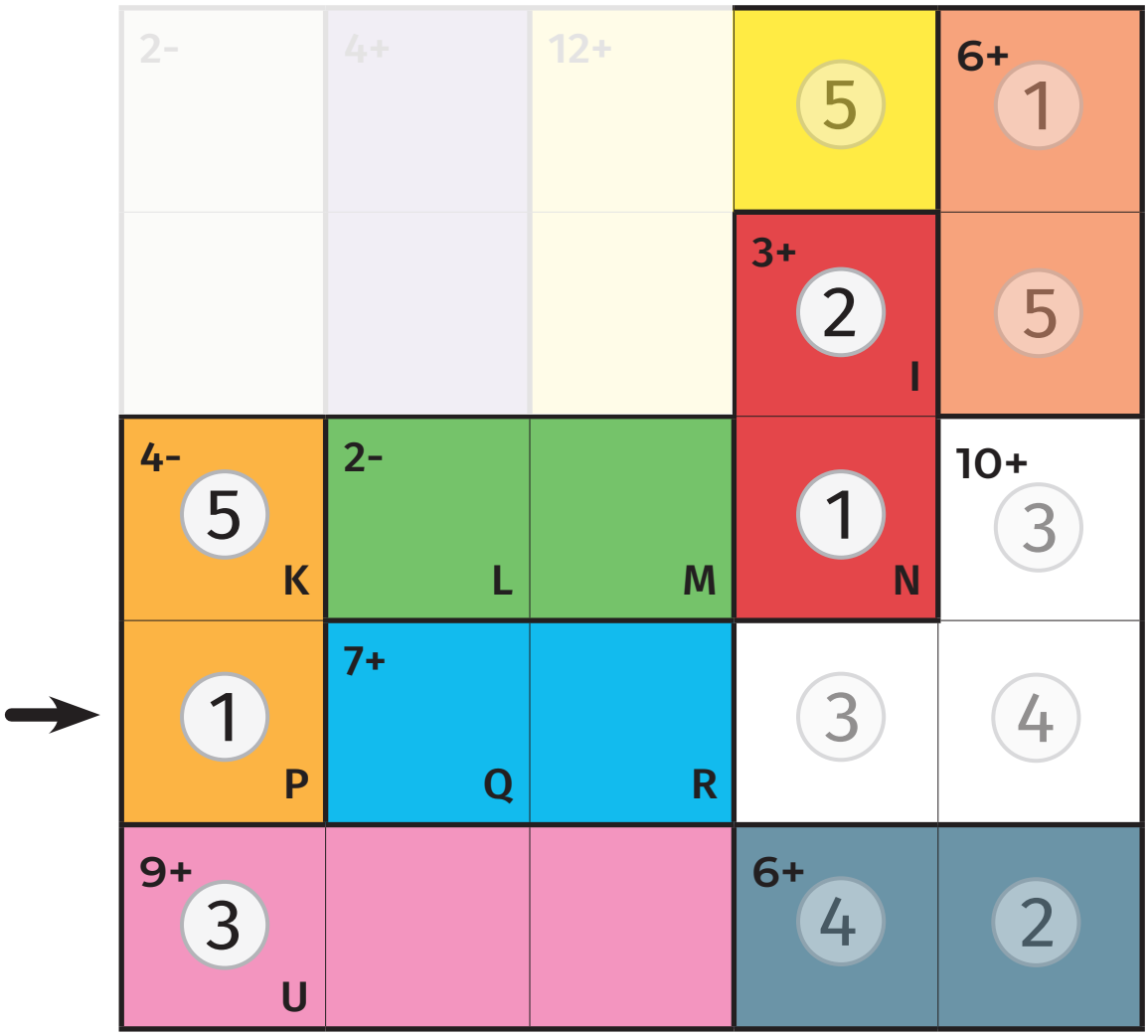
- Cell Y is 2 to complete the **grey** cage.
- In column 4, cell S (**white**) must be 3.

In column 5, to complete the **white** 'reverse L' cage, the two cells total 7.

- Possible candidate sets are {2,5} or {3,4}.
- Cannot be {2,5} due to the "No Repeats" rule.
- The digit 3 must be in cell O and 4 in cell T.

In the last column (6+) cage, cell J = 5 and cell E = 1.

Solving and Reasoning Steps - Part 2

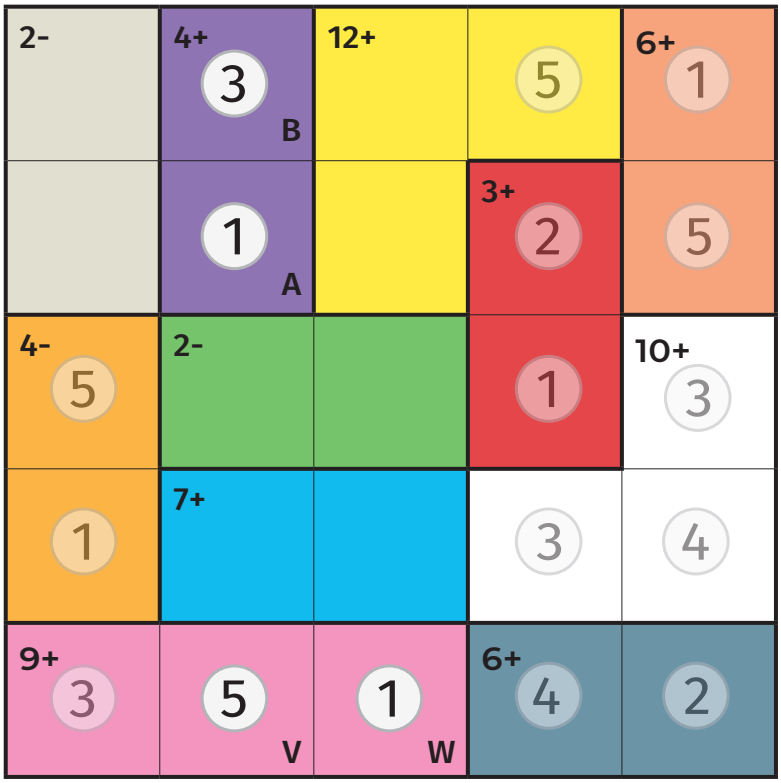


In row 4, cell P (orange) = 1, as Q + R = 7. Cell K = 5.

When there is a difference or range of 4 in a 5 x 5 grid, the numbers must be {1,5}.

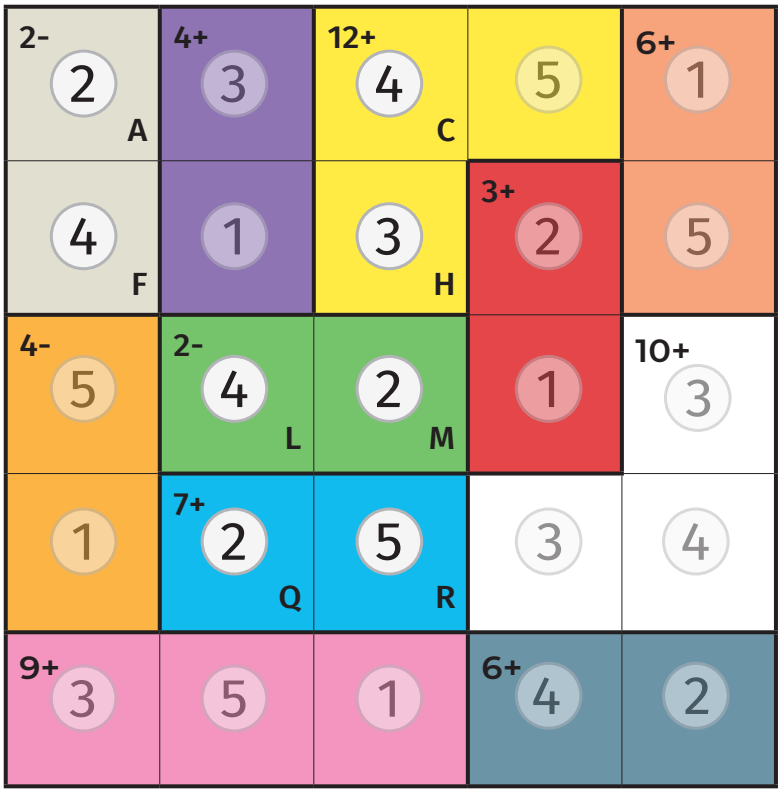
- Cell N (red) = 1, as cells L and M will be 2 or 4 to make a difference of 2.
- Similarly, cell U (pink) = 3 for the first column.

Solving and Reasoning Steps - Part 3



Cell B (purple) = 3 as it cannot be a "1" as no repeats of digits are permitted.

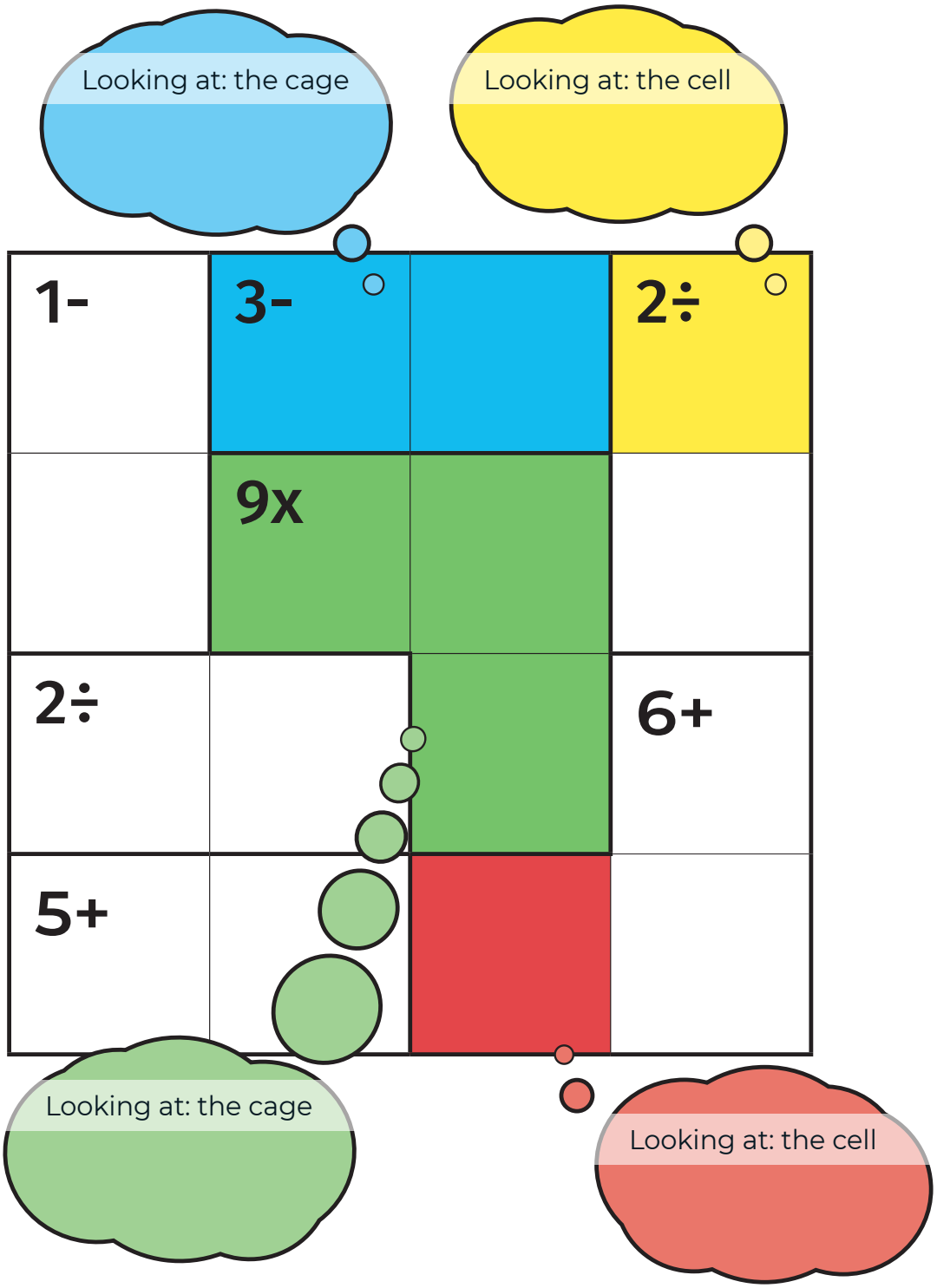
Cell V = 5, as not equal to 1, hence cell W (pink) =1.



Cell F = 4, as not equal to 2 (repeat in row) and cell A = 2.

Cell C = 4 and cell H (yellow) = 3 (last cell in row).

Cell M = 2, as not equal to 4, cell L (green) = 4 and cell Q = 2, and finally cell R (blue) = 5.



"Thinking Clouds" can be used to assess student reasoning. Students can also add in their own "Thinking Clouds" to demonstrate their reasoning ability. Dating these records will assist teachers to monitor student progress.

Possible "Thinkings":

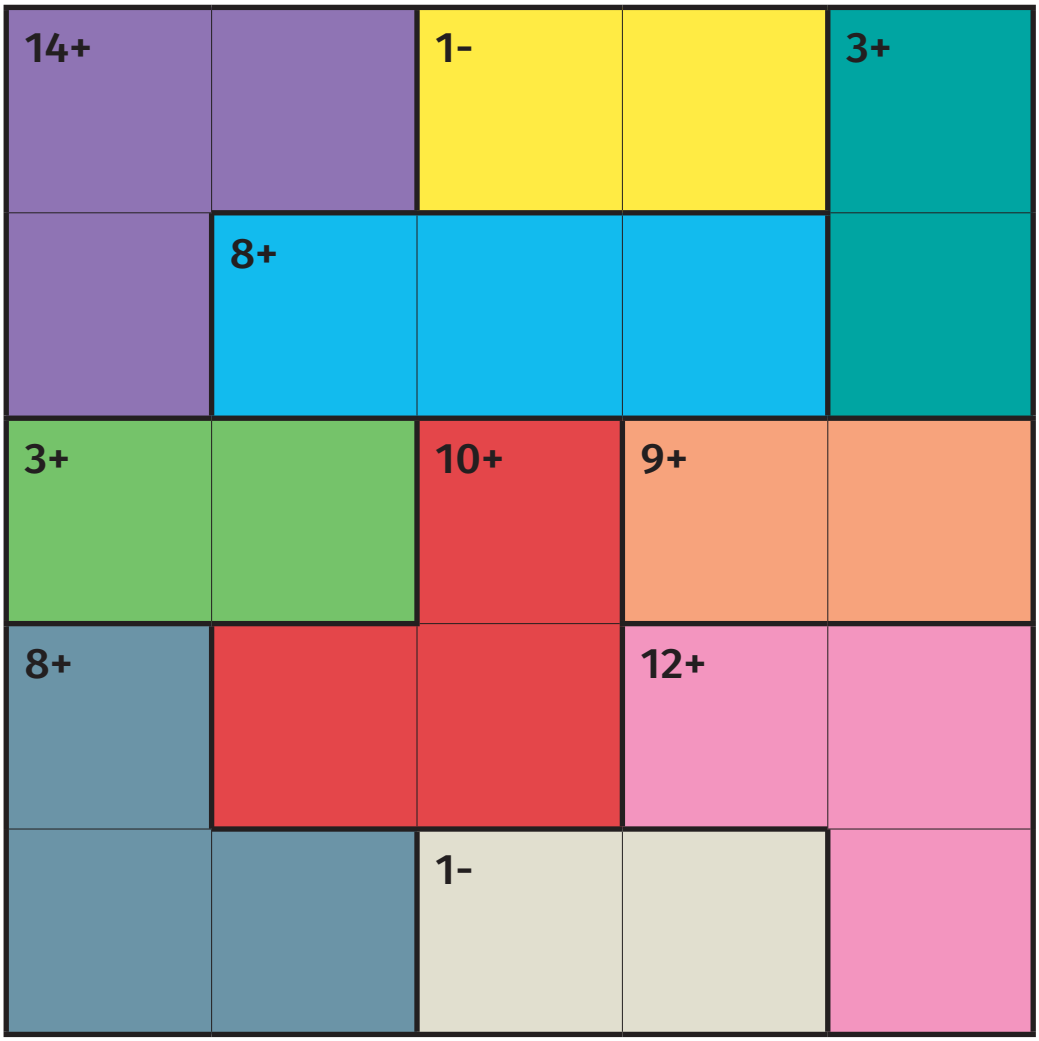
Blue Cloud: For the (3-) cage, the digits have to be {1, 4} or {4, 1}

Yellow Cloud: As the digits 1 and 4 are used in the **blue** (3-) cage, the digit cannot be 3, it must be 2.

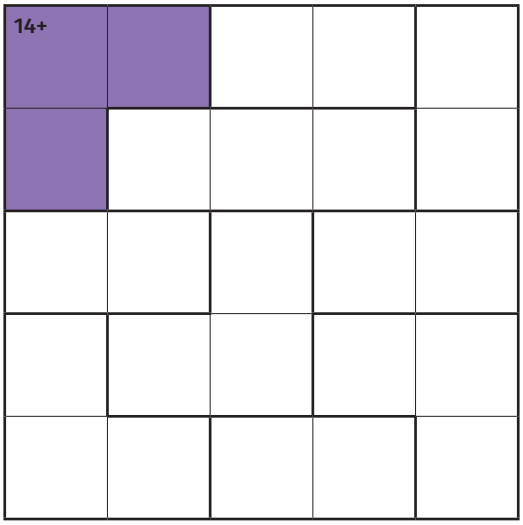
Green Cloud: The three cell (9x) cage must be {3, 1, 3} with no repeats.

Red Cloud: This cell must be 2, as it is the last remaining digit for column 3.

Addition and Subtraction 5 x 5 Medium

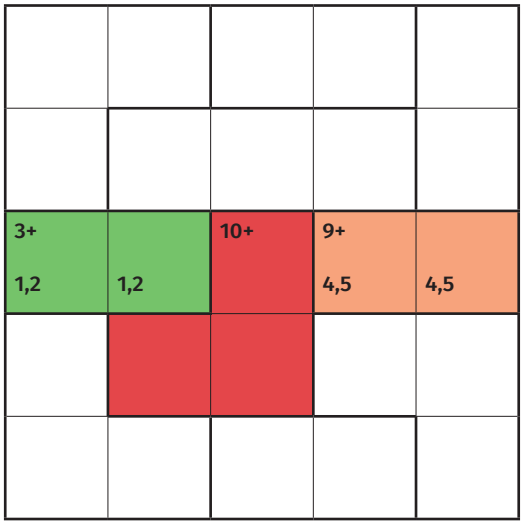


Possible Starts:



What must be in the 3 cells of the (14+) cage when using the digits 1 to 5?

Remember: No repeats.



Start at row 3.

For the (3+) cage, candidates are {1,2} in any order.

For the (9+) cage, candidates are {4,5} in any order.

Given the row sum = 15,

Then the middle red cell is 15-(9+3) = 3.

Other David Dunstan & Paul Swan Materials

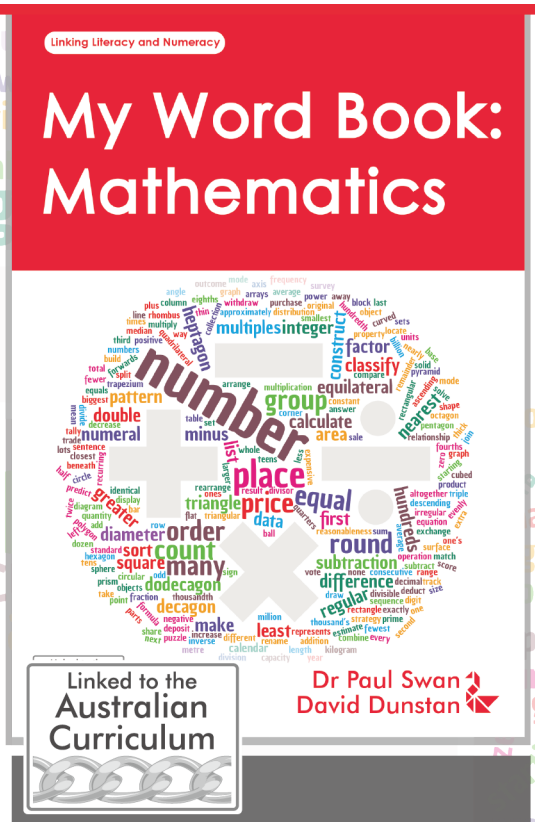
My Word Book: Mathematics by Dr Paul Swan and David Dunstan.

A comprehensive listing of mathematical languague, Featuring:

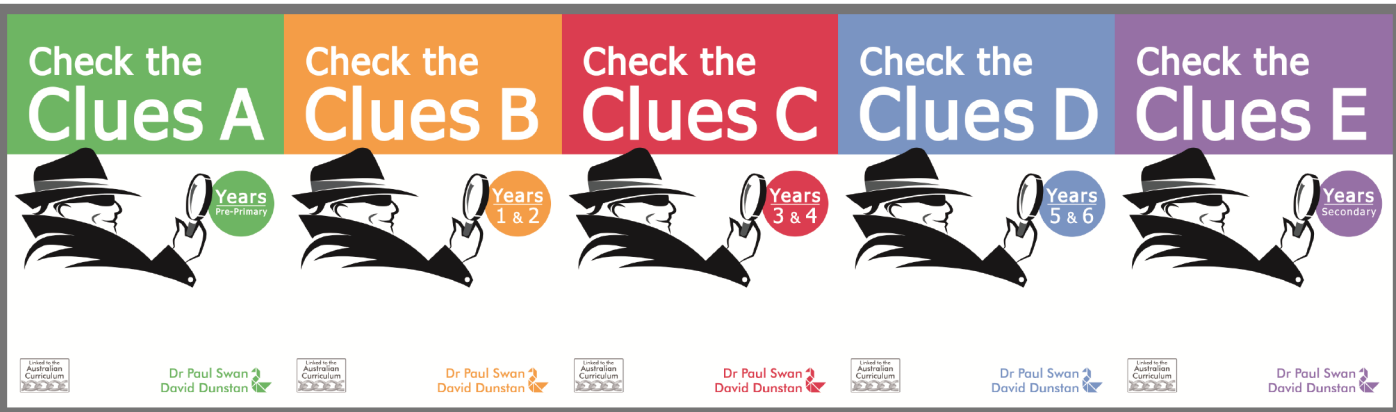
- Lists for all of the Curriculum Substrands, organised by year level.
- A focus on words students should know to be able to complete questions.
- Lists of essential vocabulary that has already appeared in NAPLAN (2010-17)

Guidance on:

- Word building (e.g. roots, suffixes).
- Troublesome "sounds-alike" words (homonyms) and context.
- Words (and symbols) with specific mathematical meanings.



Develop comprehension of word questions with



Check The Clues A, B, C, D and E by Dr Paul Swan and David Dunstan.

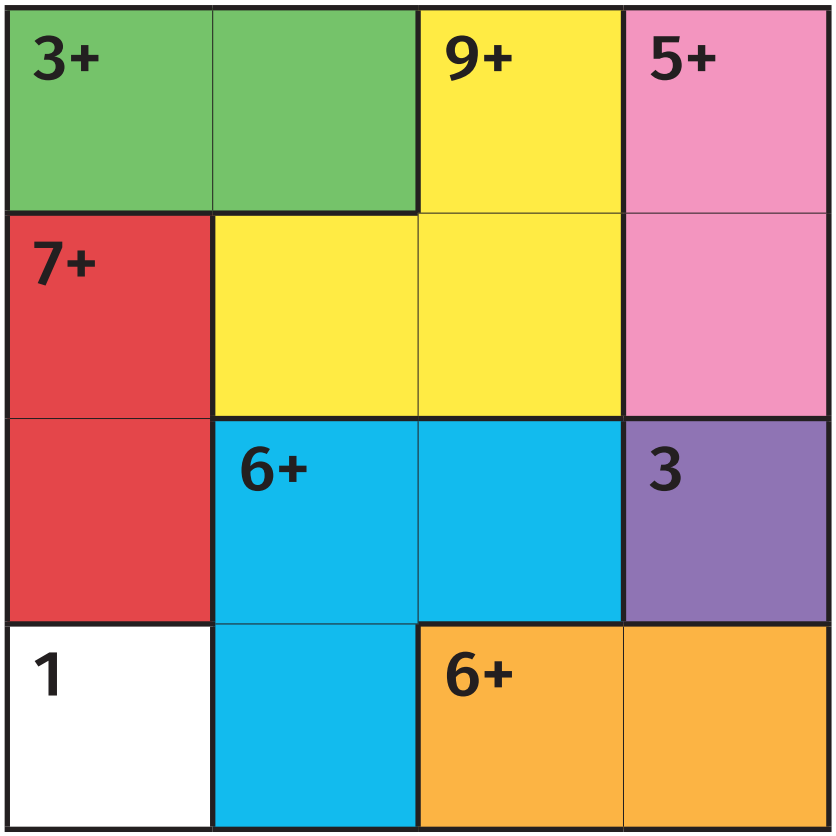
Based on ***Polya's four step approach to Problem Solving*** students work in groups of four to solve word problems.Improve mathematical literacy of your students while solving problems.

1 KenKen

KenKen - Easy

1	2	3	4
1	2	3	4
1	2	3	4
1	2	3	4

Place the numbered counters so that each cage adds up to the target number shown. No row or column can have any repeated numbers.

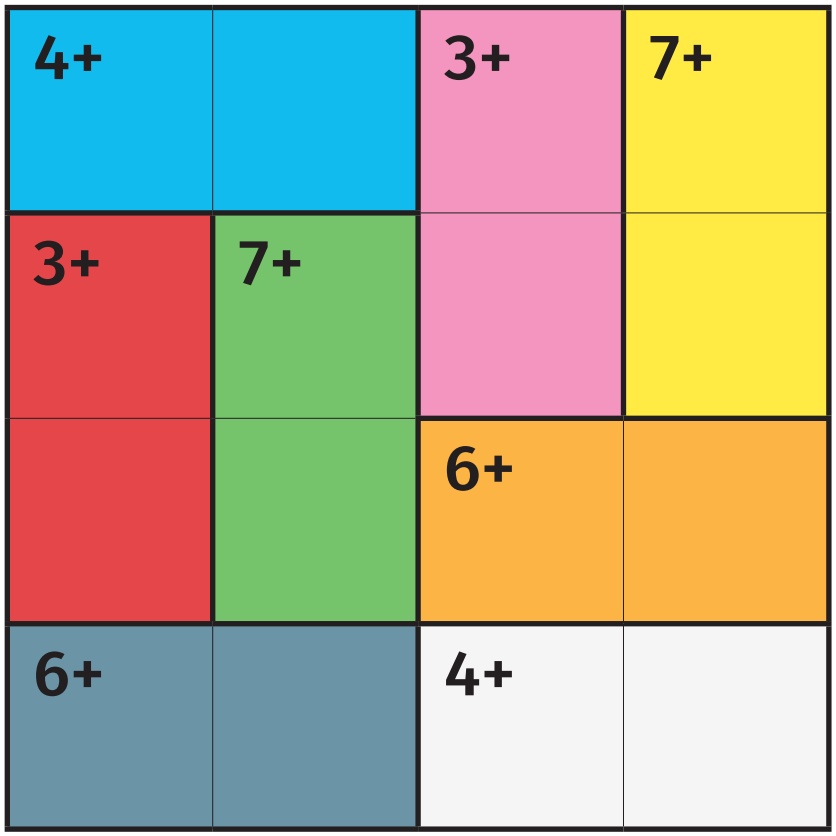


2 KenKen

KenKen - Medium

1	2	3	4
1	2	3	4
1	2	3	4
1	2	3	4

Place the numbered counters so that each cage adds up to the target number shown. No row or column can have any repeated numbers.

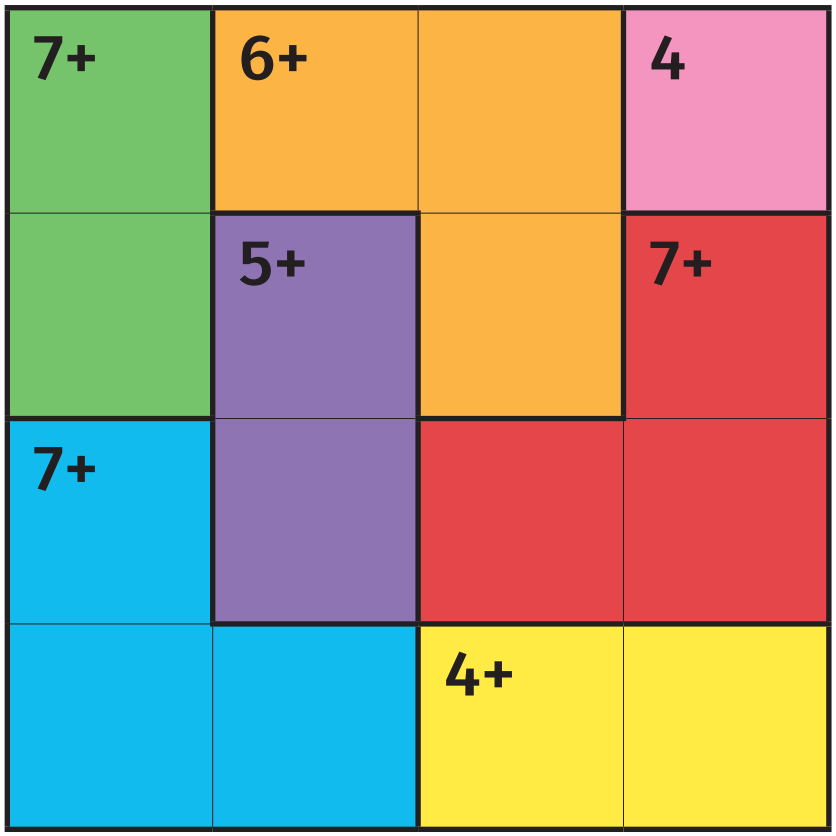


3 KenKen

KenKen - Hard

1	2	3	4
1	2	3	4
1	2	3	4
1	2	3	4

Place the numbered counters so that each cage adds up to the target number shown. No row or column can have any repeated numbers.

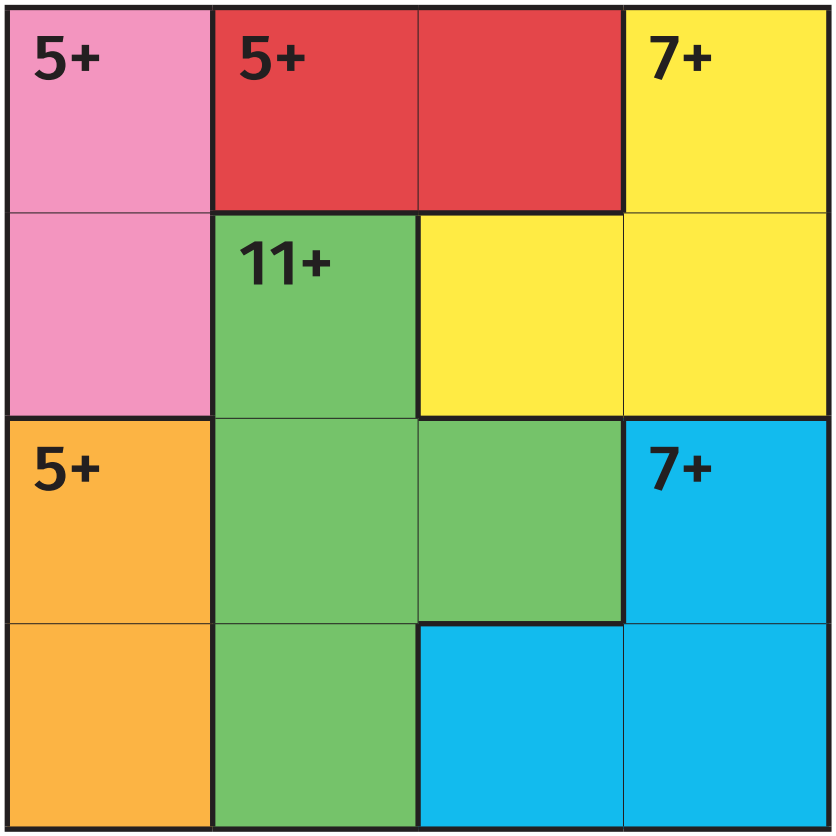


4 KenKen

KenKen - Expert

1	2	3	4
1	2	3	4
1	2	3	4
1	2	3	4

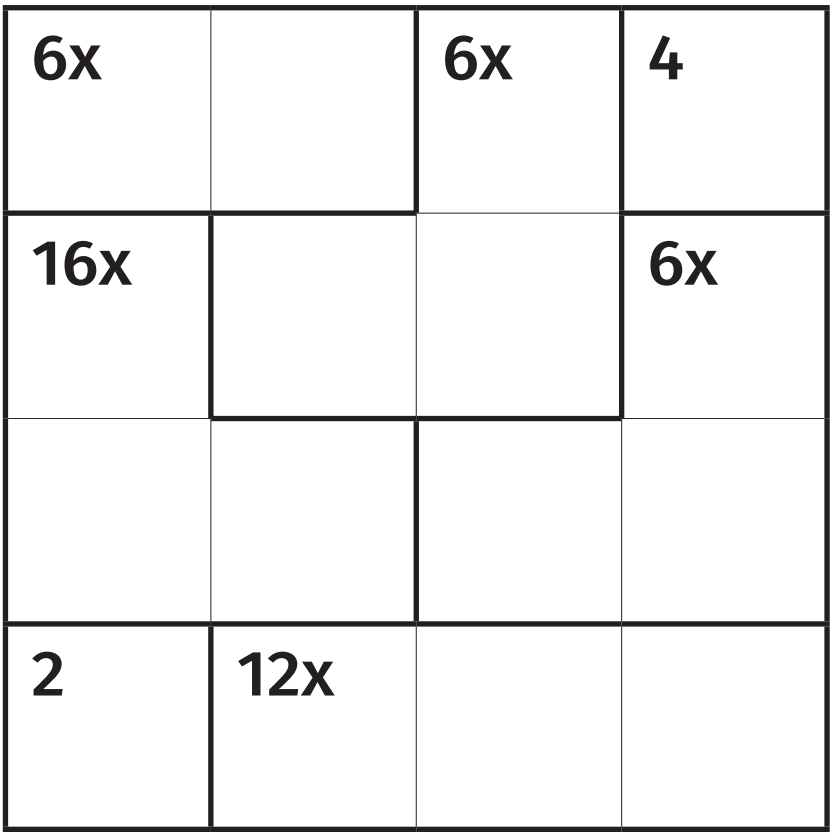
Place the numbered counters so that each cage adds up to the target number shown. No row or column can have any repeated numbers.



5 KenKen

KenKen - Easy Multiplication (4 x 4)

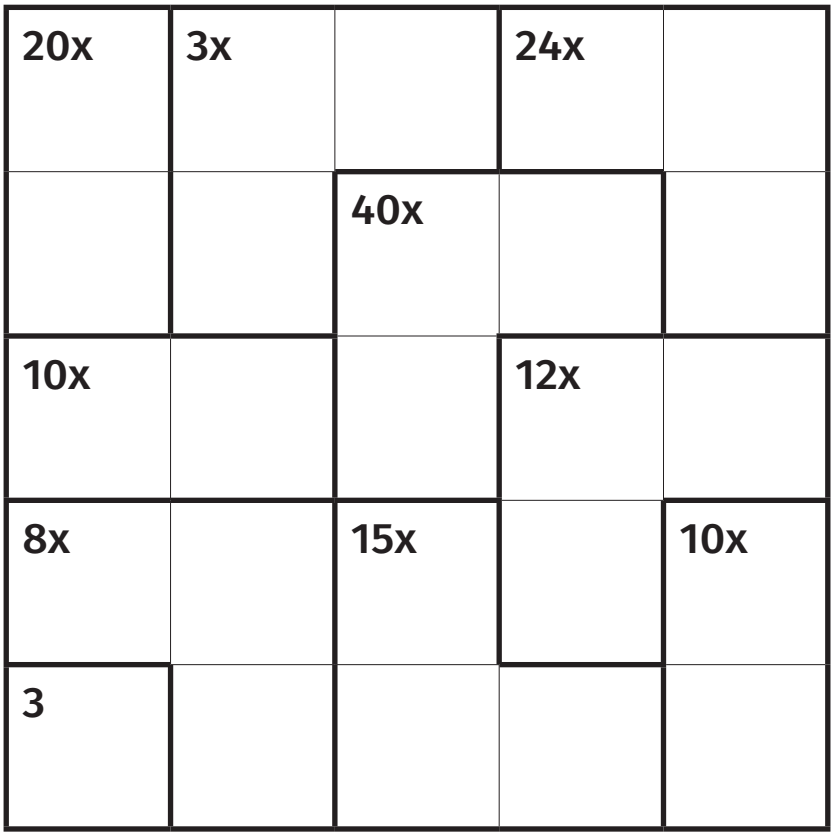
Remember, no row or column can have any repeated numbers.



6 KenKen

KenKen - Easy Multiplication (5 x 5)

Remember, no row or column can have any repeated numbers.



7 KenKen

KenKen - Easy
All Operations (5 x 5)

Remember, no row or column can have any repeated numbers.

2-		4	2÷	
8x		4-		16+
	1-			
4-		3-	5+	
7+				1

8 KenKen

KenKen - Easy
All Operations (6 x 6)

Remember, no row or column can have any repeated numbers.

3+		10x	2÷		16x
2-	15+		20x		
				1	60x
11+	4	3÷			
		5-		1-	8+
6x		2÷			

9 KenKen

KenKen - Easy Addition (8 x 8)

(Hint) Row and Column Totals:
1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 = 36

Remember, no row or column can have any repeated numbers.

12+		5+		11+	7+		14+
	7+		12+		17+		
5+	8+			10+		13+	
	3+	8+			17+	10+	
13+		14+					3+
		15+	12+	9+		5+	
20+				8+			12+
	10+				8+		

10 KenKen

KenKen - Hard Addition (6 x 6)

Remember, no row or column can have any repeated numbers.

7+	12+	8+			5+
			16+		
5+	6+	7+			13+
		6+	8+		
9+				11+	
	10+		3+		