MATHEMATICAL NOVELTIES

Practical activities designed to motivate the teaching and learning of number.

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Copies of this book may be obtained from:

ISBN 0 646 19424 0

First Published 1994

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The author wishes to thank Karen Anderson, Alan Sadler and Theo Wieman for their constructive criticism and support.

See also Number Novelties by the same author published 1993

Typeset & Design by J.Swan
Calculator Artwork by Norm Ferguson
Printed by Success Print

MATHEMATICAL NOVELTIES

Practical activities designed to motivate the teaching and learning of mathematics.

Preface

All of us at one time or another have heard the cry of anguish or seen the despondent look on a child's face when we announce it is time for maths. Let's face it, Mathematics is probably the most hated subject in school – and yet it doesn't have to be.

This book is designed for teachers who are looking for ways to motivate their students and for all those children who hate mathematics. Inside you will find tricks and puzzles designed to be used by teachers with their students or by students on their own. Teachers will also find some Mathematical Magic designed to liven up any maths lesson.

The activities are always fun, provide a challenge and often call for considerable calculation, thereby providing children with painless practice. More importantly, the solutions often depend on the properties of numbers, investigation of which may lead to a greater understanding of mathematical principles.

More able students will often appreciate *why* a trick works once they understand *how* it works. Many of the Number Novelties may be solved by the application of simple algebra, providing an informal introduction to algebra for lower secondary students.

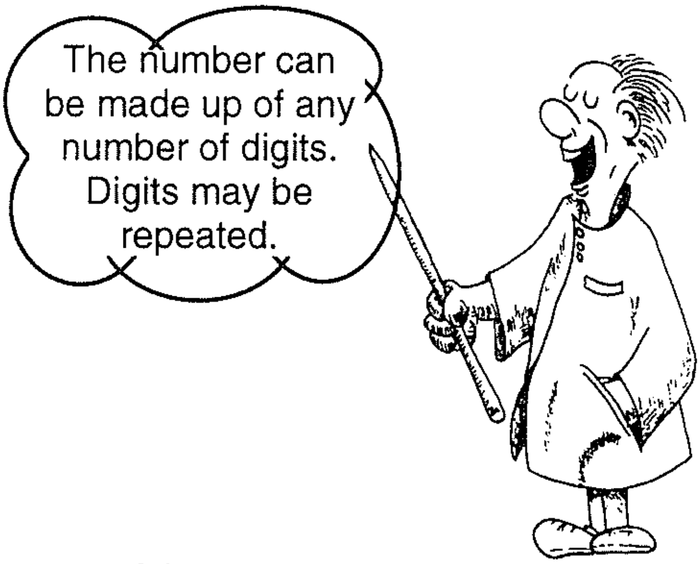
Students will be amazed by the Mathematical Magic tricks contained in the book. Teachers will be grateful for the answers and detailed explanations given.

Mathematical Novelties will help children see the magic of mathematics and help them appreciate its power and beauty.

Contents

TEN TIMES

- Choose a number at random. **4 751**

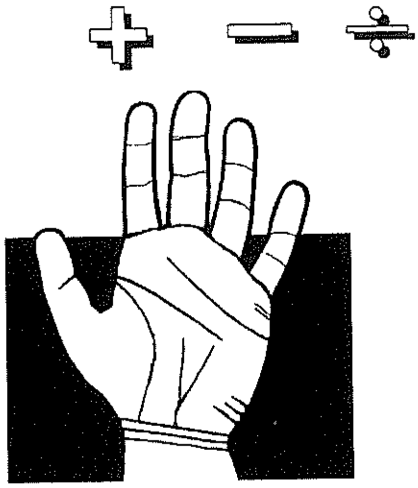


- Multiply by **10.** **47 510**
- Subtract your original number from your new number.

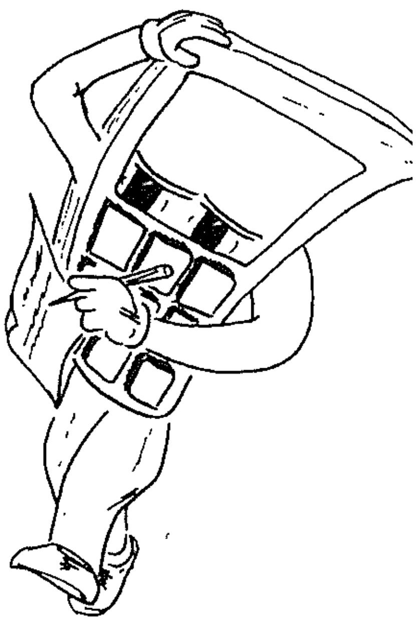
$$\begin{array}{r} 47\,510 \\ -\ 4\,751 \\ \hline =42\,759 \end{array}$$

- Divide this answer by your original number. **42 759 ÷ 4 751 = ?**
- Try again using some different starting numbers.
- Try some single digit, two digit, three digit and four digit numbers.
- Write about what you notice.
- Try to explain why it happens.

GIVE ME FIVE



- ☞ Choose any number. **e.g. 23**
- ☞ Add the next counting number after this on to the original number. **23 + 24 = 47**
- ☞ Add **9.** **47 + 9 = 56**
- ☞ Divide by **2.** **28**
- ☞ Subtract your original number. **28 – 23 = ?**
- ☞ Try again using some different starting numbers.



- ☞ Try using a single digit number.
- ☞ Try using a three digit number.
- ☞ Write about what you notice.
- ☞ Try to explain why it happens.

ROLE REVERSAL

- Choose any two digit numbers where no digit is repeated. **e.g. 63**
- Reverse the digits. **36**
- Subtract the smaller number from the larger number.

$$\begin{array}{r} 63 \\ -\ \underline{36} \\ =\ \underline{27} \end{array}$$

- Repeat the process using the new number that is formed.

$$\begin{array}{r} 72 \\ -\ \underline{27} \\ =\ \underline{45} \end{array}$$

- Continue until you reach a single digit answer.

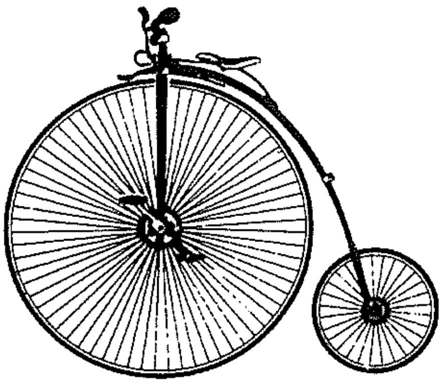
$$\begin{array}{r} 54 \\ -\ \underline{45} \\ =\ ? \end{array}$$

Try these:

65 (1 step)	71 (2 steps)	93 (2 steps)
74 (3 steps)	26 (4 steps)	57 (5 steps)

What answer do you always reach?

Try to find a pattern to determine whether a starting number will only take one step to reach an answer.



CYCLING DIGITS

- Write down a four digit number. **e.g. 9 351**
- Cycle the digits by moving the digit in the thousands place to the hundreds place. The digit in the hundreds place is then moved to the tens place. The tens digit is moved to the units place. The units digit is then moved to the thousands place. **1 935**
- Continue cycling the digits until four numbers have been produced.

5 193
3 519

- Add the four numbers together.

9 351 + 1 935 + 5 193 + 3 519 = 19 998

- Find the sum of the original four digits. **9 + 3 + 5 + 1 = 18**
- Divide the total from adding the four numbers by the sum of the four digits. **19 998 ÷ 18 = ?**

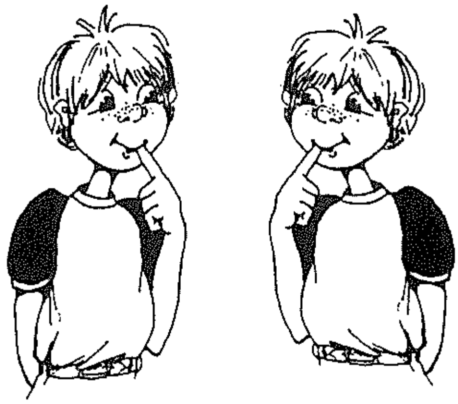
Try starting with other four digit numbers.

Write about what you notice.

Try starting with a three digit number or a five digit number.

What happens if you begin with a number where all four digits are the same?

DOUBLE UP



- Enter a 3 digit number into your calculator. **e.g. 267**
- Multiply by **7.** **1 869**
- Multiply by **11.** **20 559**
- Multiply by **13.** **???**

What happened? Discuss with your neighbour.

Try using some other 3 digit numbers.

Try using 3 digit numbers where all the digits are the same

e.g. 222, 555, 888.

Try using numbers with trailing zeros.

e.g. 100, 400, 700.

Write about what you notice.

Does the order in which you multiply matter?

Try multiplying by 13, then 11 and then 7.

What happens?

Try using a different order.

Try to explain why it happens.

DO OR DIE

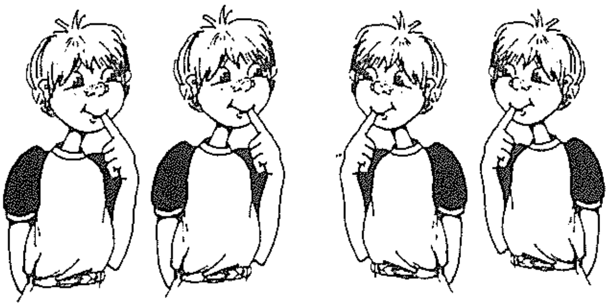
- Throw two dice and note the numbers on the top of the dice. **e.g. 3, 5**
- Multiply these two numbers. **3 x 5 = 15**
- Lift the dice and note the two numbers on the bottom of the dice. **4, 2**
- Multiply these two numbers. **4 x 2 = 8**
- Multiply the top number of the first die by the bottom number of the second die. **3 x 2 = 6**
- Multiply the top number of the second die by the bottom number of the first die. **5 x 4 = 20**
- Add your four results. **15 + 8 + 6 + 20 = ?**

Roll the dice again and repeat the procedure.

Try a few more times.

- Discuss with your neighbour.

Write about what you notice.



DOUBLE TROUBLE

- Write down any three different single digit numbers. **e.g. 3, 4, 7**
- Use these three numbers to make nine 2 digit numbers. Digits may be repeated.

33,	**44,**	**77**
34,	**43,**	**73**
37,	**47,**	**74**

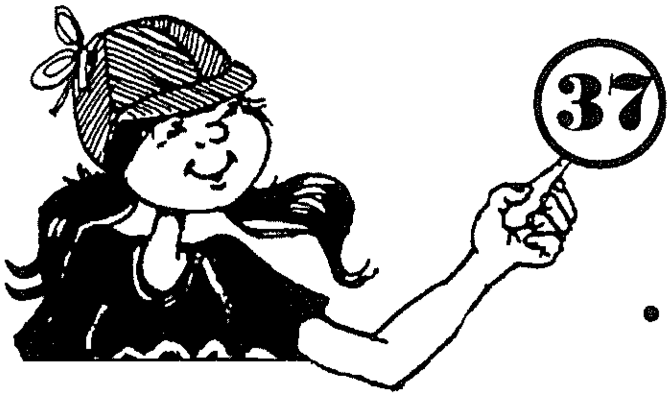
- Add the nine 2 digit numbers.

33 + 44 + 77 + 34 + 43 + 73 + 37 + 47 + 74 = 462

- Add the original three numbers **3 + 4 + 7 = 14**
- Divide the larger number by the smaller number. **462 ÷ 14**
- Write down your answer.
- Try this again using three different single digit numbers.
- Try starting with three odd numbers and then three even numbers.
- Discuss with your neighbour.
- Try using consecutive single digit numbers.

1, 2, 3	2, 3, 4	3, 4, 5	4, 5, 6
5, 6, 7	6, 7, 8	7, 8, 9	

Write about what you notice.



THIRTY SEVEN

- Try the following multiplications:

3 x 37 =	4 x 37 =	5 x 37 =
6 x 37 =	7 x 37 =	8 x 37 =
9 x 37 =	10 x 37 =	11 x 37 =
12 x 37 =	13 x 37 =	14 x 37 =

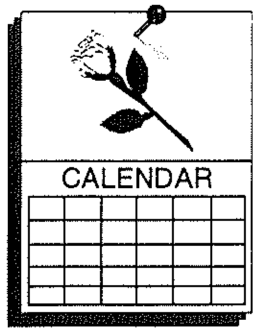
- Try to predict what 15 x 37 equals. Use any patterns you have noticed to predict what 18 x 37, 21 x 37 and 24 x 37 equal.
- Try to explain why this pattern occurs.
- Choose a three digit multiple of 37 from any of the numbers you have obtained so far and write it down. **e.g. 518**
- Move the hundreds digit to the units place and the ten's digit to the hundreds place and the unit digit to the tens place to make a new number. **185**
- Repeat. **851**
- Divide each of the two new numbers by 37. **e.g. 185 ÷ 37 = ?**
  851 ÷ 37 = ?

Did they divide exactly?

- Try again using some different three digit multiples of 37.
- Write about what you notice.

What happens if the three numbers (ie 518, 185 and 851) are added and then divided by 37?

WHAT DAY WAS IT?



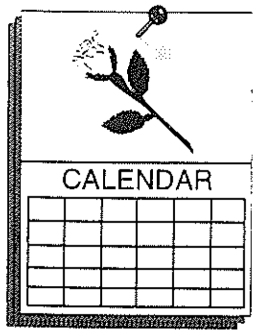
Judy was born on January 17th, 1981.
Which day of the week was that?

- To determine the day of the week on which you were born use the following steps.
- Write down the last two digits of the year in which you were born. **81**
- Divide this number by 4.
  Don't worry about any remainders. **20**
- Find the number for the month in which you were born from the table below. **January = 1**
- Write down the date in the month when you were born. **17**
- Add all the numbers from the previous steps **81 + 20 + 1 + 17 = 119**
- Divide this sum by 7 and note the remainder. **119 ÷ 7 = 17 r 0**
- Find the remainder in the table below to determine the day on which you were born.

MONTHS	
January	1
January (leap year)	0
February	4
February (leap year)	3
March	4
April	0
May	2
June	5
July	0
August	3
September	6
October	1
November	4
December	6

Remainder	Day
1	Sunday
2	Monday
3	Tuesday
4	Wednesday
5	Thursday
6	Friday
0	Saturday

CALENDAR MAGIC



- Choose any month from the year and draw a 3 x 3 box around any nine dates.

2001

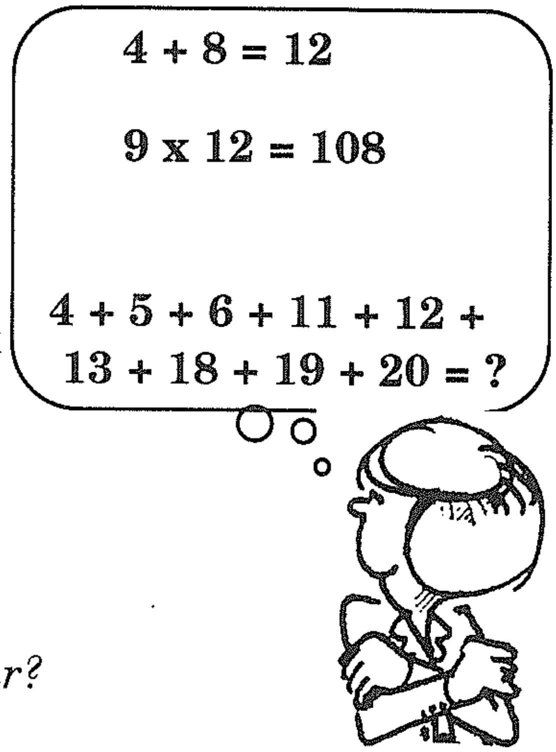
JANUARY

S	M	T	W	T	F	S
	1	2	3	4	5	6
7	8	9	10	11	12	13
14	15	16	17	18	19	20
21	22	23	24	25	26	27
28	29	30	31			

- Choose the smallest number in the box.
- Add eight to it.
- Multiply the result by nine.

What is your answer?

- Add the nine numbers in the box

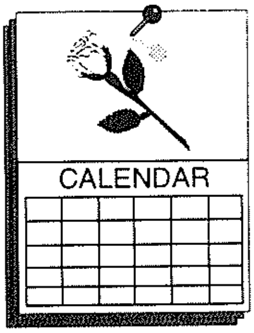


How does your answer compare to the sum of the numbers in the box?

Try some other months.

Does it work for any month in the year?

COOL CALENDAR



- Choose any month from the calendar.
- Find four dates that form a 2 x 2 pattern.
- Draw a box around them.

2001

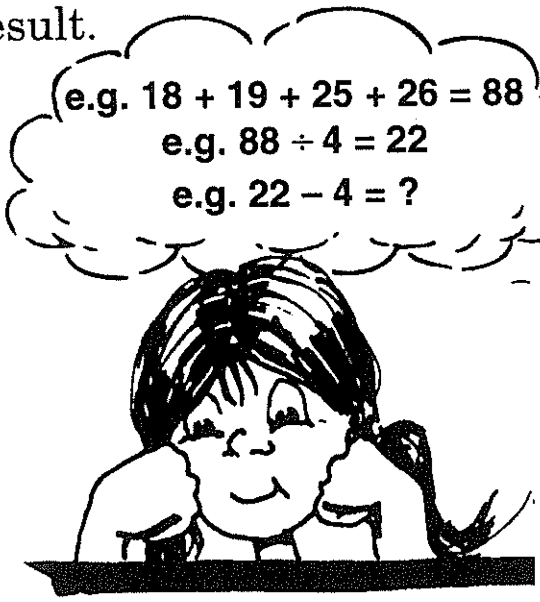
JANUARY

S	M	T	W	T	F	S
	1	2	3	4	5	6
7	8	9	10	11	12	13
14	15	16	17	18	19	20
21	22	23	24	25	26	27
28	29	30	31			

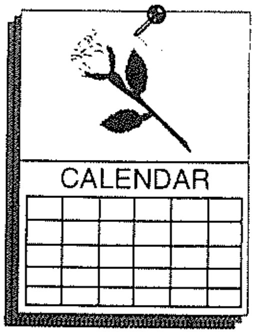
- Add all the four numbers together.
- Divide the answer by four.
- Then subtract four from this result.

Discuss what you notice with your neighbour

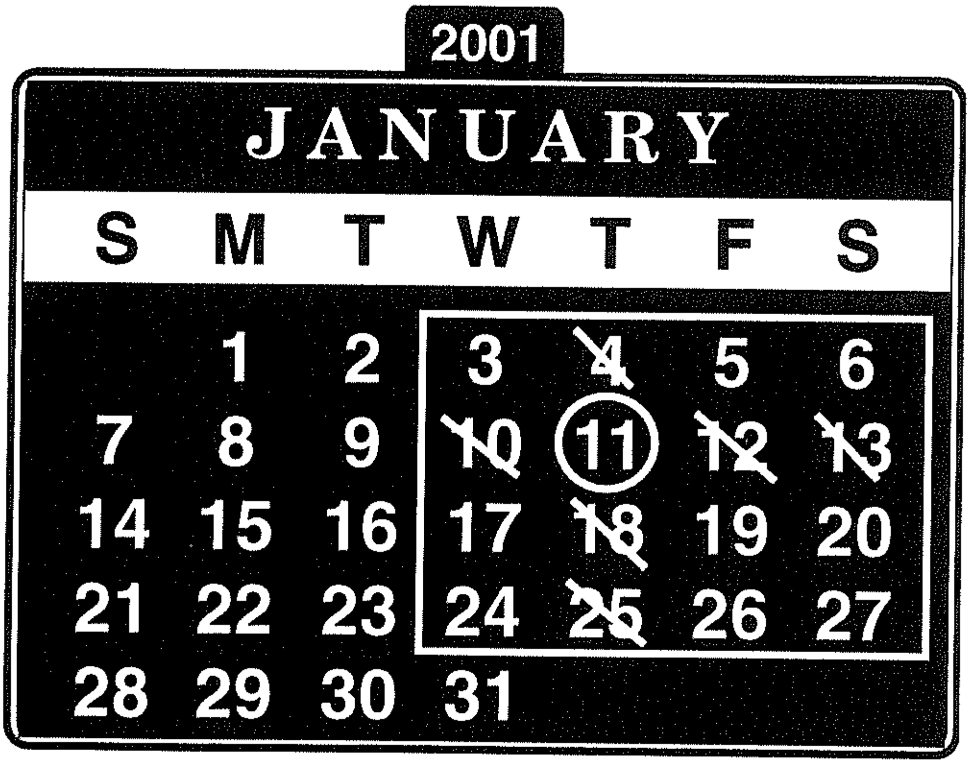
- Try some other dates.
- Try some other months.
- Write about what you notice.



CALENDAR CAPER



- Choose any month from the calendar.
- Draw a 4 x 4 box around a block of dates.
- Circle any date within the block.
- Cross out all the numbers that are in the same row or column as the circled number.



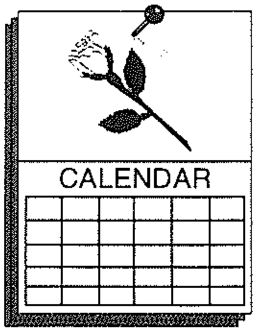
- Circle another number and cross off all the other numbers in the same row and column.
- Repeat a third time.
- There should now only be one date left. Draw a circle around this date.
- Add all the circled dates. Note the total.
- Try the same procedure using the same set of dates.

What do you notice?

- Repeat the entire procedure twice using another set of dates from another month.

Write about what happens.

CRAZY CALENDAR



Choose any month from the calendar and draw a 4 x 4 box around any 16 dates.

- Add the numbers in each of the four corners.

e.g. 3 + 6 + 24 + 27 = 60

- Draw a circle around any number in the box.
- Draw another circle around a different number in the box, but make sure it is not in the same row or column as the first number.
- Choose another number that is in a different row or column to the other circled numbers.
- Choose one more number that is not in the same row or column as any of the other circled numbers.

JANUARY

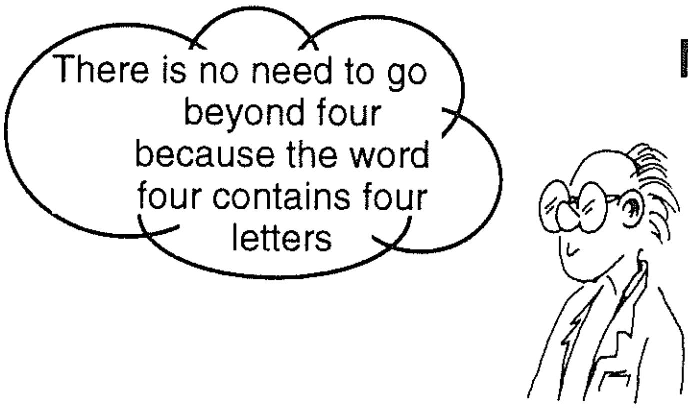
S	M	T	W	T	F	S
	1	2	3	4	5	6
7	8	9	10	11	12	13
14	15	16	17	18	19	20
21	22	23	24	25	26	27
28	29	30	31			

- Add your 4 circled numbers. **e.g. 4 + 10 + 19 + 27.**
- Compare this total with your total for adding the 4 corner numbers.
- Try again using the same set of dates.
- Try using a different set of 16 dates from the same month.
- Try using 16 dates from a different month.
- Write about what you notice.

NUMBER WORDS

Try the following puzzle.

- Choose a number. **e.g. 54**
- Write it in words. **Fifty Four**
- Count the letters. **9**
- Write it as a word. **Nine**
- Count the letters. **4**

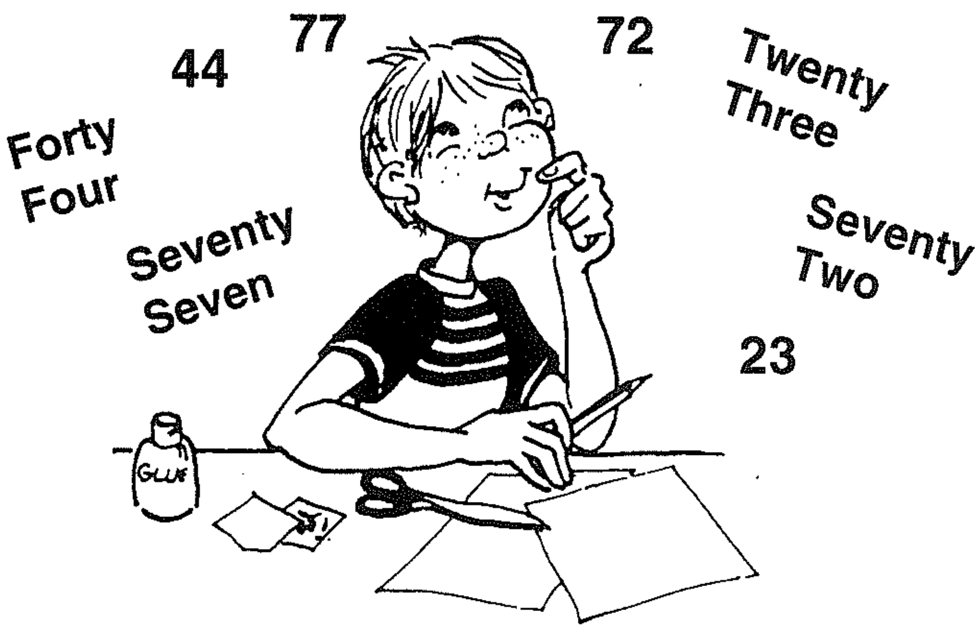


Try starting with the following numbers and see what happens.

44, 72, 77, 23

Discuss what happened with your neighbour.

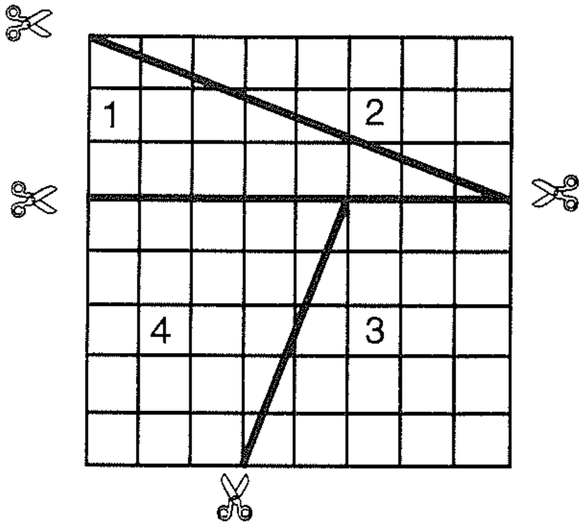
Choose some starting numbers of your own and note what happens when you follow the above procedure.



EXPANDING AREA

✎ Using centimetre grid paper, cut out a square of side 8cm.

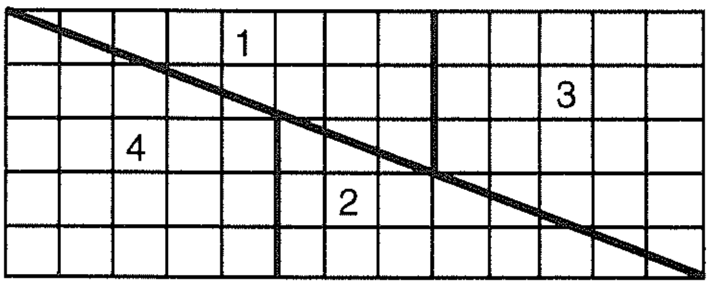
✎ Find the area of this square.



(Diagram not full size)

✎ Draw lines as shown on the diagram and cut along these lines.

✎ Rearrange the pieces as shown in the second diagram.



(Diagram not full size)

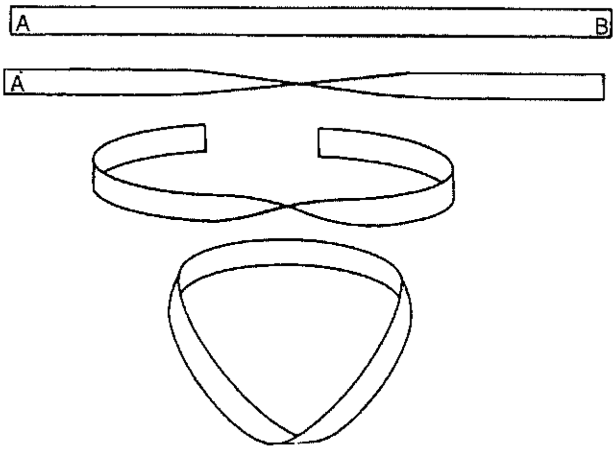
✎ Find the area of the rectangle that is formed.

What has happened?

A SHAPE WITH A TWIST

Moebius Bands were first investigated by German astronomer and mathematician August Ferdinand Moebius (1790 - 1868).

To make your own Moebius Band follow the instructions.



- Take a strip of paper about 30cm long and 2 cm wide. Mark one end **A** and the **back** of the other end **B**.

- Form a half-twist in the strip by turning one end over. i.e. through 180°.

- Join the ends to form a shape. This is called called a Moebius Band.

- Starting at **A** draw a line around the band until **B** is reached and continue on until you return to **A**.

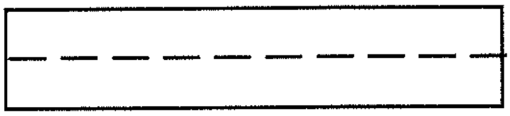
Did the line need to cross an edge?

Does the Moebius Band have one or two sides?

- Try creating a Moebius Band using a full twist.

A SHAPE WITH A TWIST II

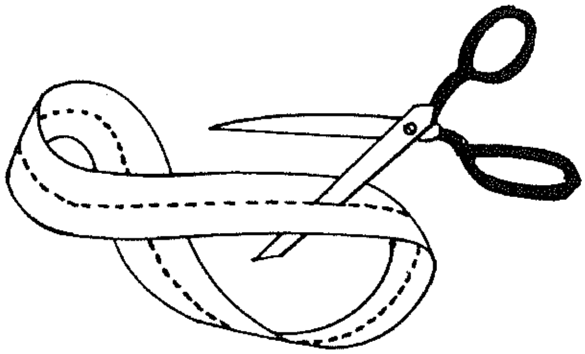
SPLIT STRIPS



Cut a new strip and mark a line down the centre before giving your strip a half-twist and joining the ends together.

Next, cut along the centre line.

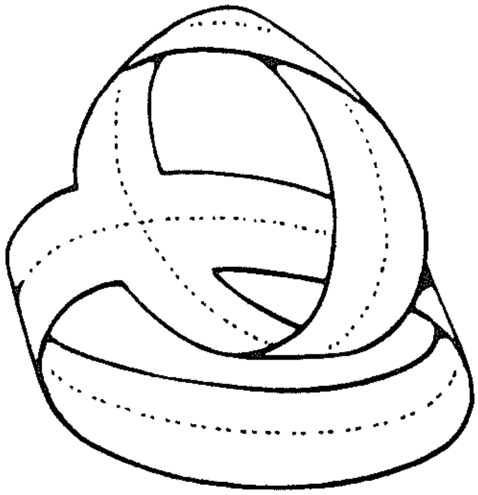
What happens?



Try marking two lines along a strip and repeating the process.

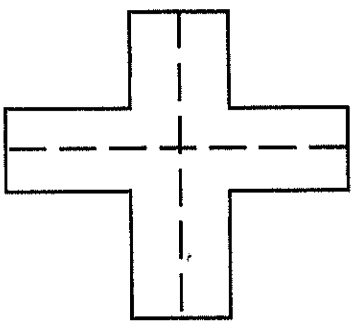
What happens?

Try using a Moebius band with a full twist.



The model shown above was made from a large paper cross. To construct it, join 2 opposite arms to form a cylinder. Twist the other two arms and join to form a Moebius Strip.

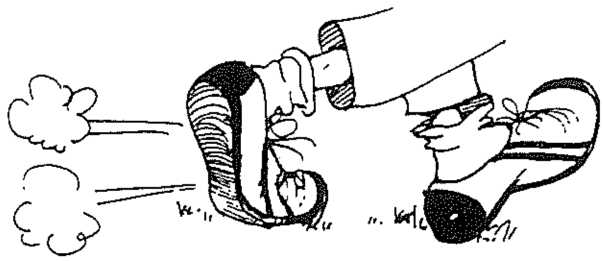
For a variation:



First draw a line down the centre of each arm of the cross, then join as above. Now cut along the lines (two cuts will be necessary).

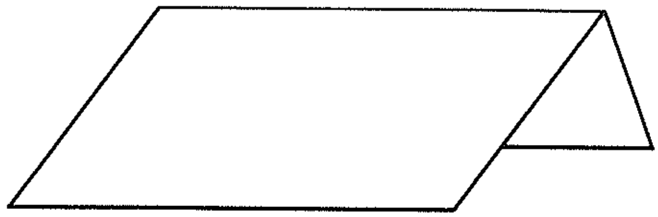
The result should surprise you.

STEPPING THROUGH PAPER

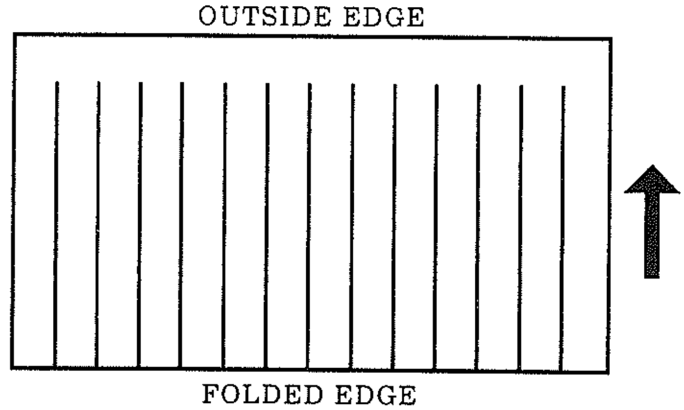


You will need a sheet of A4 paper and a pair of scissors.

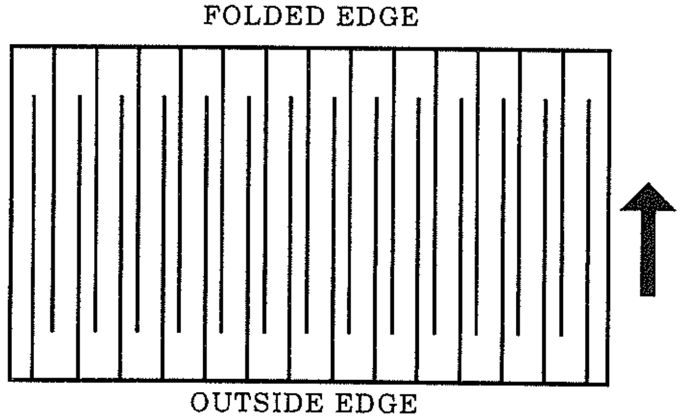
Fold a sheet of A4 paper in half lengthwise.



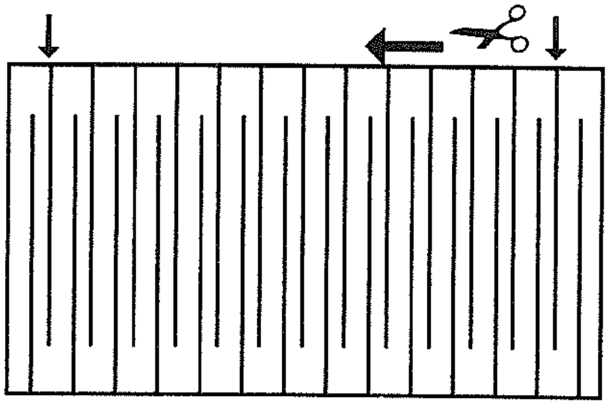
Cut from the folded edge down toward the outside of the paper. Make sure the cuts are *at least* 2 cm apart.

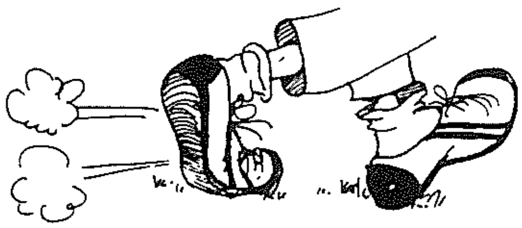


Next cut back from the outside edge toward the fold. Make sure you cut between your first set of cuts and that you stop short of the fold.



Use the scissors to cut along the fold in the paper between the two marked points.





Open the paper and step through.

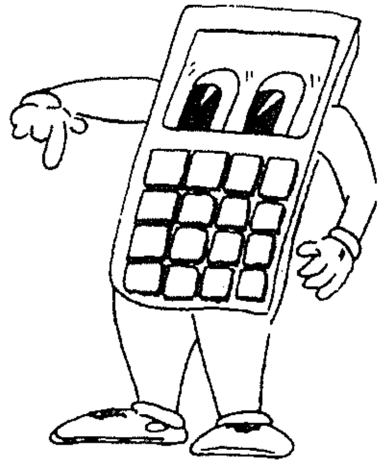
CALCULATION SHORTCUTS

SQUARING ANY TWO DIGIT NUMBER.

Squaring or multiplying a two digit number by itself may be made easier using the following shortcut.

Decrease or increase to the nearest multiple of ten.

Compensate by adding or subtracting the same amount to this number.



$$24^2 = (20 \times 28) + 4^2$$

In this case 4 was subtracted

So 4 was added

$$= 560 + 16$$
$$= 576$$

Normally we would multiply 24 x 24 which would involve a number of steps. If the unit digit of one of the numbers becomes zero then the number of calculations required to complete the multiplication is reduced, thereby making the calculation easier to do mentally.

In the above example 20 x 28 is easier to calculate than 24 x 24. Finding the square of a single digit number and adding it to the total is fairly simple.

Try using the shortcut method.

(a) 23^2	(b) 32^2	(c) 35^2
(d) 41^2	(e) 43^2	(f) 49^2
(g) 52^2	(h) 67^2	(i) 94^2

Check with your calculator.

CALCULATION SHORTCUTS

X

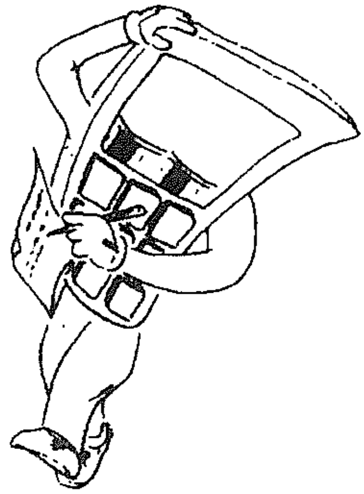
MULTIPLYING RELATED NUMBERS

Consider two numbers like 43 and 37, which are both 3 away from 40. The following shortcut may be used to make the multiplication a little easier.

$$\begin{aligned} 43 \times 37 &= (40 + 3)(40 - 3) \\ &= 40^2 - 3^2 \\ &= 1\,600 - 9 \\ &= 1\,591 \end{aligned}$$

To use this shortcut both numbers need to be the same amount away from a number that is easy to square mentally. e.g. a two digit multiple of 10 or a three digit multiple of 100.

$$\begin{aligned} \text{e.g. } 597 \times 603 &= (600 - 3)(600 + 3) \\ &= 600^2 - 3^2 \\ &= 360\,000 - 9 \\ &= 359\,991 \end{aligned}$$



Try these using the shortcut method.

(a) 33 x 27	(b) 31 x 29	(c) 44 x 36
(d) 42 x 38	(e) 51 x 49	(f) 53 x 47
(g) 68 x 72	(h) 398 x 402	(i) 797 x 803

Check with your calculator.

CALCULATION SHORTCUTS

÷ 5

DIVIDING BY 5

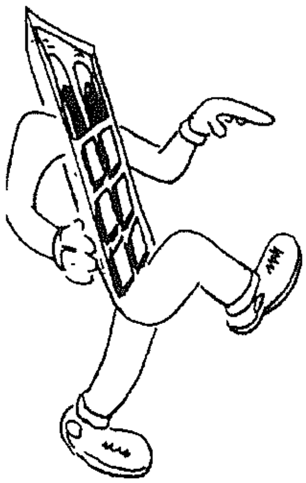
A quick way to divide a number by five is to double the number and cross off the units digit if it is a zero or write it as a decimal.

e.g. **735 ÷ 5 → 147~~0~~ → 147**
336 ÷ 5 → 672 → 67·2

Try dividing the following numbers by 5 using the method outlined above.

(a) 275 (b) 414 (c) 665 (d) 491 (e) 304

Check with your calculator.



x 15

MULTIPLYING BY 15

A quick method of multiplying a number by fifteen is to add zero on to the end of the number and then halve the number and add the two numbers together.

e.g. 486 x 15 →	**4860**	(adding zero to the end)
	+ 2430	(halving the above number)
	= 7290	

Try multiplying the following numbers by 15 using the method outlined above.

(a) 342 (b) 416 (c) 548 (d) 643 (e) 781

Check with your calculator.

CASTING OUT NINES

Complete the nine times table.

1 x 9 = 9	6 x 9 = ______
2 x 9 = 18	7 x 9 = ______
3 x 9 = ______	8 x 9 = ______
4 x 9 = ______	9 x 9 = ______
5 x 9 = ______	10 x 9 = ______

Add the two digits in each answer.
Write about what you notice.

This fact is the basis of the "casting out nines" method of checking calculations.

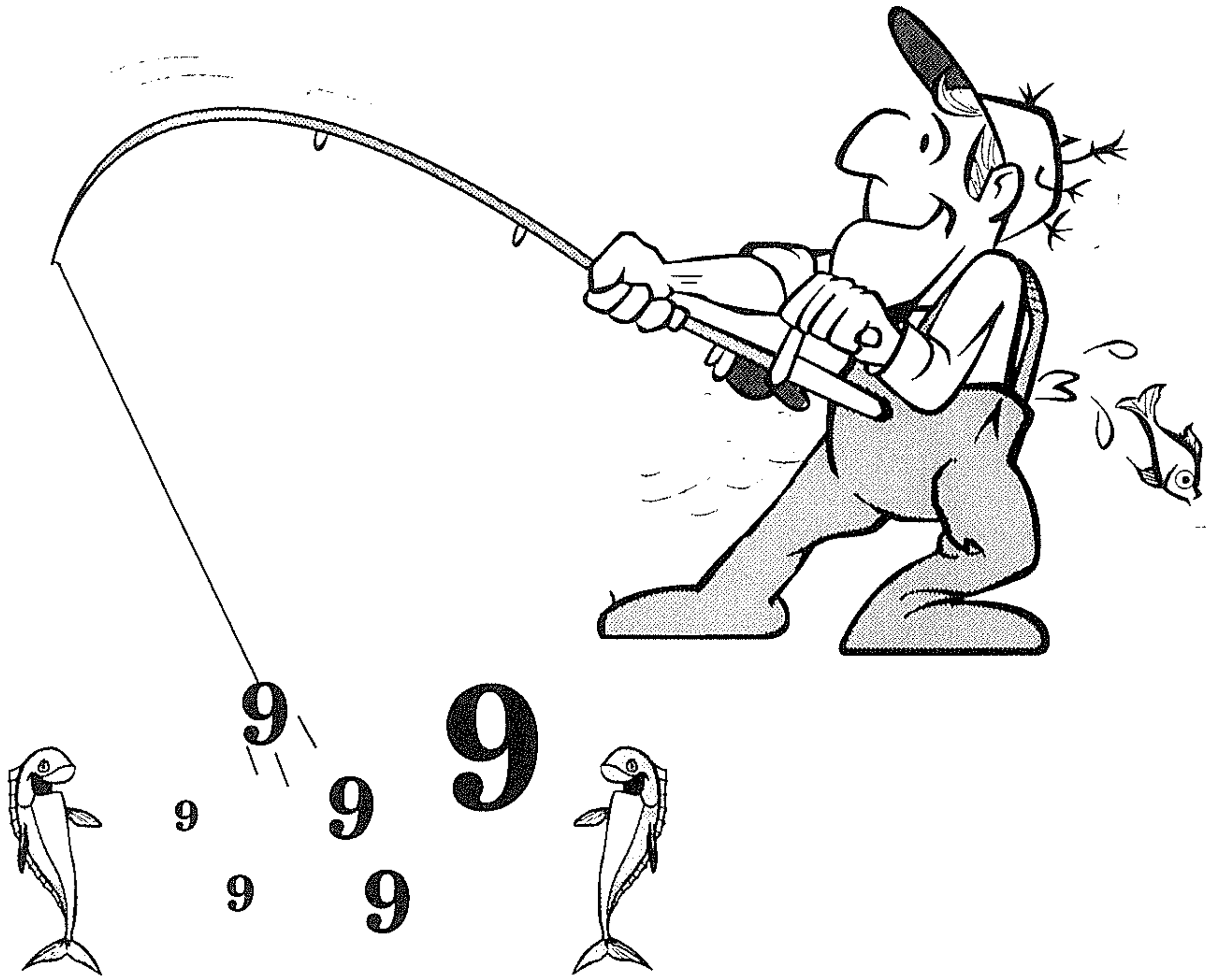
If a number is exactly divisible by 9 (i.e. there is no remainder) then its digits should add up to 9.

e.g. 891 is exactly divisible by 9 because
$8 + 9 + 1 \rightarrow 18 \rightarrow 1 + 8 \rightarrow 9$.

687 is not divisible by 9 because
$6 + 8 + 7 \rightarrow 21 \rightarrow 2 + 1 \rightarrow 3$.

Try dividing 9 into 687.

Write down the remainder.



Use the “casting out method to determine whether the following numbers are evenly divisible by 9. If they are not evenly divisible by 9 then write down the value when all the digits are added.

(a) 657	(b) 848	(c) 514	(d) 748
(e) 432	(f) 329	(g) 666	(h) 1018

If the number was not evenly divisible by 9 then divide it by 9 carefully noting the remainder.

Compare the remainder to the number produced by using the casting out nines method.

Write about what you notice.

CALCULATION CHECKS

There are several ways to check an addition sum such as:

$$\begin{array}{r} 357 \\ +\ \underline{174} \\ =\ \underline{531} \end{array}$$

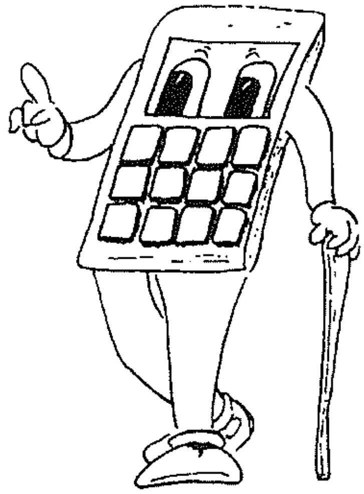
One way would be to use a calculator.
Another way would be to reverse the calculation and add.

$$\begin{array}{r} 174 \\ +\ \underline{357} \\ =\ \underline{531} \end{array}$$

You could subtract one number from the total to check.

$$\begin{array}{r} 531 \\ -\ \underline{357} \\ =\ \underline{174} \end{array} \quad \text{or} \quad \begin{array}{r} 531 \\ -\ \underline{174} \\ =\ \underline{357} \end{array}$$

or you could try casting out nines.

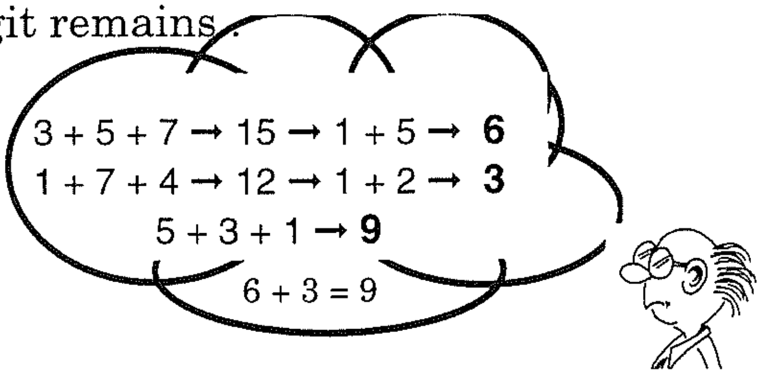


To check a calculation such as

$$\begin{array}{r} 531 \\ -\ \underline{357} \\ =\ \underline{174} \end{array}$$

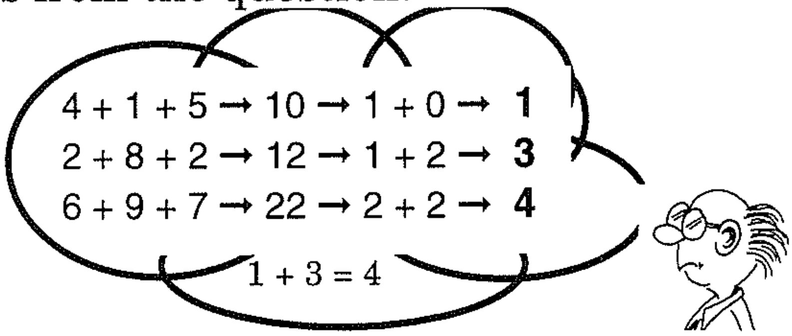
we would need to determine the remainder of each number in the calculation when it is divided by 9. It would take some time to divide all the numbers by nine. A quick method of obtaining the remainders when dividing by nine is to add the digits in the number until a single digit remains.

	357	→	15	→	6	
+	174	→	12	→	3	
=	531	→	9			



If the answer is correct the remainder of the answer will equal the sum of the remainders from the question.

e.g.	415	→	10	→	1
	+ 282	→	12	→	3
	= 697	→	22	→	4



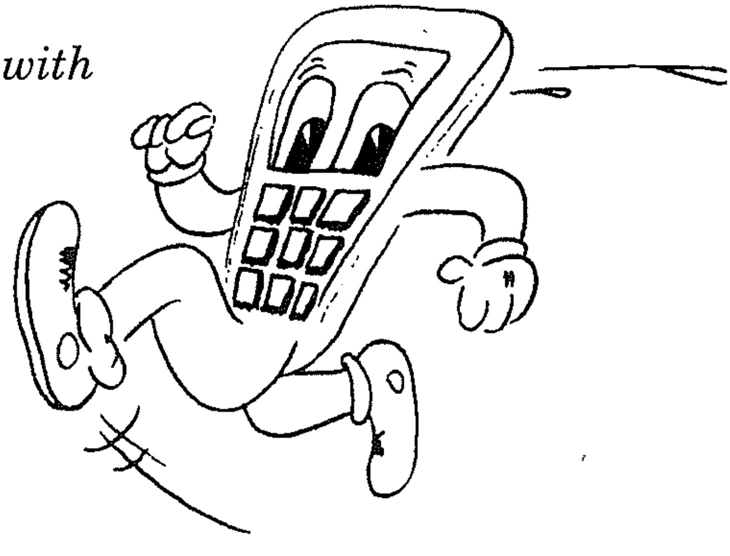
Use the "casting out nines" method to check whether the following problems are right or wrong.

(a)	527 + 258 = 775	(b)	144 + 717 = 861	(c)	462 + 349 = 801
(d)	371 + 243 = 614	(e)*	576 + 248 = 914	(f)	256 + 732 = 888

Try making up some of your own.

Will casting out nines work with subtraction?

Try it and see.



CALCULATION CHECKS

The "casting out nines" method of checking a calculation works with subtraction as well as addition but this time you compare the value from the larger numbers with the values from the smaller numbers. For example:

$$\begin{array}{rcccc} 4361 & \rightarrow & 14 & \rightarrow & \textcircled{5} \\ -\ 2744 & \rightarrow & 17 & \rightarrow & 8 \\ =\ 1617 & \rightarrow & 15 & \rightarrow & 6 \end{array}$$

$4 + 3 + 6 + 1 \rightarrow 14 \rightarrow 1 + 4 \rightarrow \mathbf{5}$

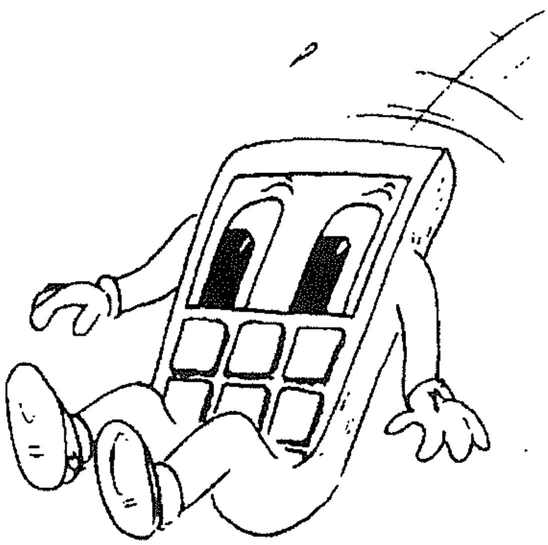
$8 + 6 \rightarrow 14 \rightarrow \textcircled{5}$

If the two circled values are the same then the answer is probably correct.

Try using this technique to check these calculations.

(a) $\begin{array}{r} 517 \\ -\ 76 \\ \hline =\ 441 \end{array}$ (b) $\begin{array}{r} 676 \\ -\ 368 \\ \hline =\ 318 \end{array}$ (c) $\begin{array}{r} 873 \\ -\ 491 \\ \hline =\ 372 \end{array}$

(d) $\begin{array}{r} 2417 \\ -\ 936 \\ \hline =\ 1481 \end{array}$ (e) $\begin{array}{r} 4635 \\ -\ 2998 \\ \hline =\ 1647 \end{array}$ (f) $\begin{array}{r} 1078 \\ -\ 959 \\ \hline =\ 109 \end{array}$



CALCULATION CHECKS

X

The "casting out nines" method of checking a calculation may be used on a multiplication problem.

e.g.

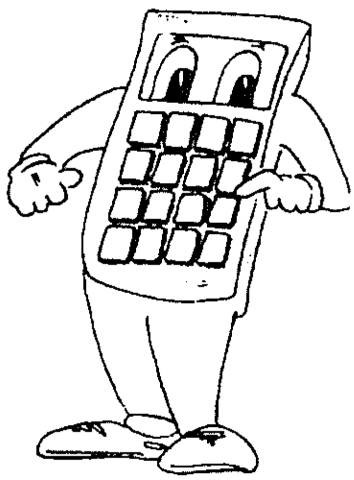
```
     32 →  5 }
  x  14 →  5 } 5 x 5 = 25 → 2 + 5 → (7)
    448 → 16 → (7)
```

The two values match therefore the answer is probably correct.

The remainders are multiplied, and a value is found by casting out nines. If this value and the remainder of the answer are the same, then the calculation is probably correct.

e.g.

```
     71 →  8            }
  x  89 → 17 →  8       } 8 x 8 = 64 → 6 + 4 → 10 → 1
   6319 → 19 → 10 → 1
```



Try checking these using the casting out nines method.

(a)	(b)	(c)
37	54	91
x 16	x 36	x 62
= 592	= 1934	= 5662

(d)	(e)	(f)
67	83	87
x 73	x 67	x 34
= 4891	= 5561	= 2858

CALCULATION CHECKS

The "casting out nines" method may also be used to check a division sum.

For example consider a division without a remainder:

38	(Cast out nines for answer)	
13)494	→ 3 + 8 = 11 → 1 + 1	= 2
	(Cast out nines for divisor)	
	13 → 1 + 3 →	= 4
Multiply	2 x 4	= 8
	(Cast out nines for the dividend)	
	494 → 4 + 9 + 4 = 17 → 1 + 7	= 8

If the two match then you are probably right.

If the division sum has a remainder then follow this procedure:

38 r 7	(cast out nines for answer)	
13)501	3 + 8 = 11→ 1 + 1	= 2
	(cast out nines for divisor)	
	1 + 3 →	= 4
Multiply	2 x 4	= 8
Add remainder		+ 7
		15
	(Cast out nines) 1 + 5	= 6
	(Cast out nines for the dividend)	
	501 → 5 + 0 + 1	= 6

If the two match then you are probably right.

Try these:

(a) 7) 266 = 38

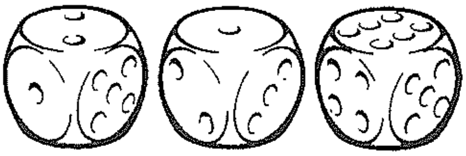
(b) 8) 536 = 66

(c) 6) 506 = 84 r 3

(d) 12) 1116 = 93

(e) 13) 728 = 59

(f) 14) 1096 = 77 r 12

DICE DROPPING 

- Drop three dice.

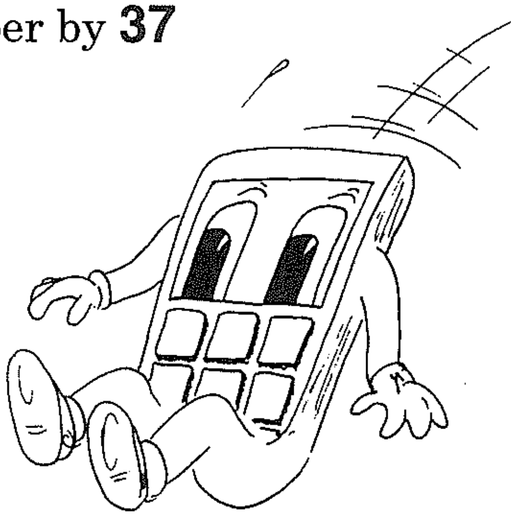
6	2	4

- Enter the three digit number produced into a calculator. The order of the numbers is not important. **624**

- Enter the numbers shown on the bottom of the dice in the same order. **624 153**

- Divide the 6 digit number by **37** and then **16 869**

- Divide by **3** **5623**



- Subtract **7** **5616**

- Divide by **9** **???**

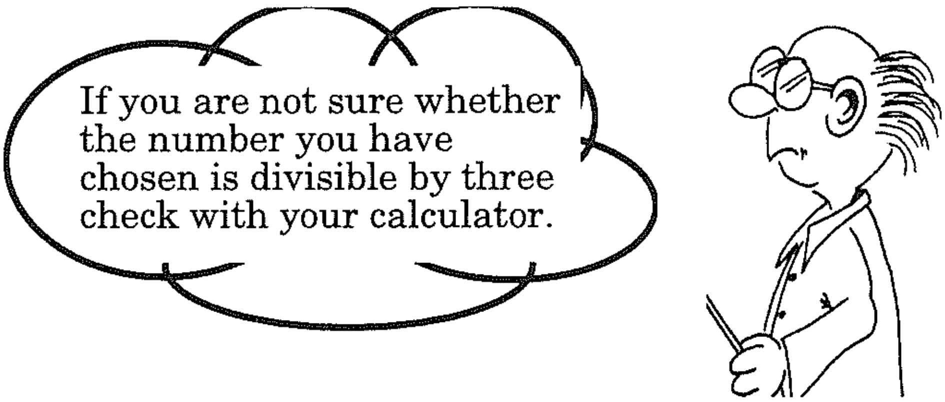
What do you notice about the result?

Try again and note what happens.

Try to explain why it works.

DIVISION DECISION I

- Choose a two digit number that is **not** divisible by 3 and enter it into your calculator **61**



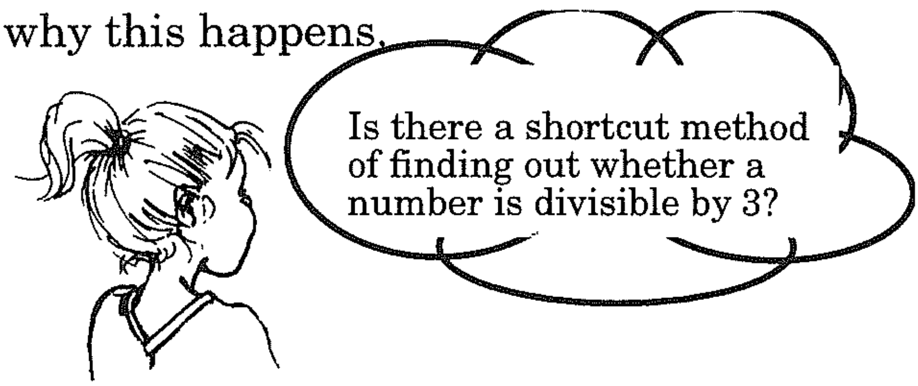
- Repeat your two digit character twice so that a six digit number is formed **616 161**

- Divide the 6 digit number by **3** **616 161 ÷ 3 = ???**

- Write about what you notice.

- Try again with some other two digit numbers that are not divisible by 3

- Try to explain why this happens.

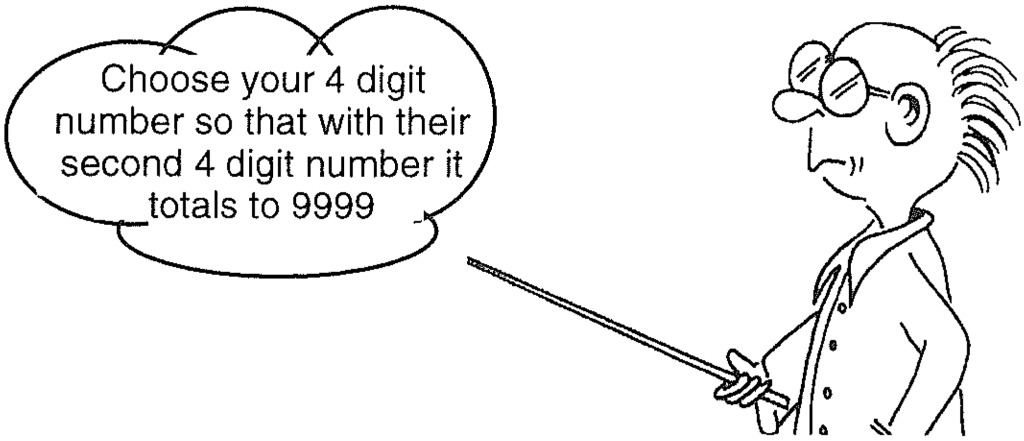


CLASSROOM MAGIC

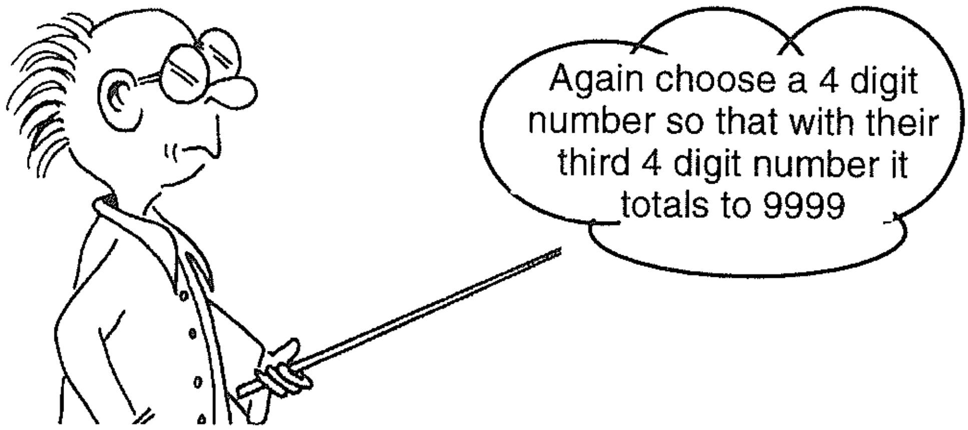
The ideas in this section require input from the teacher. A few props and a theatrical approach will greatly enhance the motivational value of each idea.
The material is this section may not be reproduced.

IT ALL ADDS UP

- Ask your student(s) to choose a 4 digit number. **e.g. 3712**
- Write it on the board
- Ask the student to choose another 4 digit number. **4653**
- You choose a 4 digit number and write it on the board **5346**



- Ask the student to choose another 4 digit number. **6719**
- You choose a final four digit number. **3280**



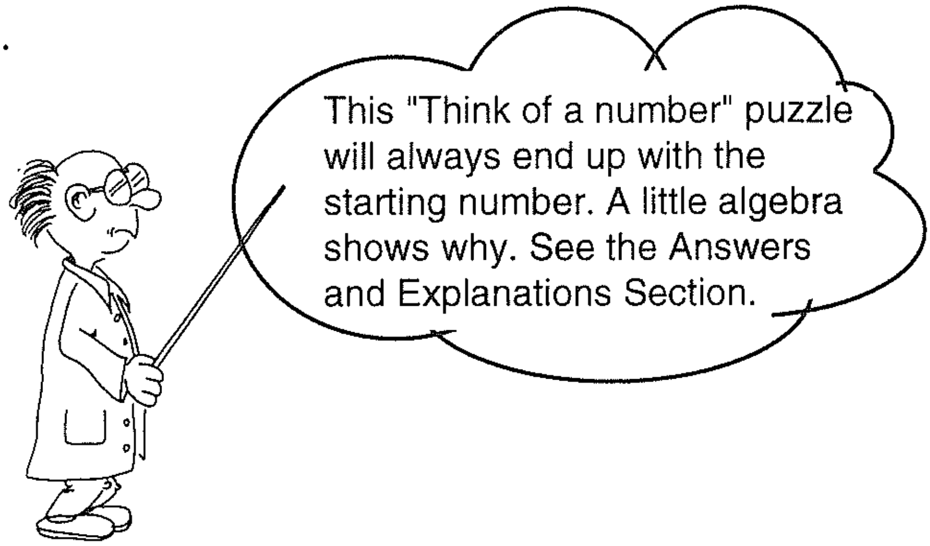
- Ask the student to add the five numbers.

While they are working out the sum, simply place 2 in front of the original number. e.g. **2**3 712 and then take 2 away from the unit's digit e.g. 23 710. This will give the answer much quicker than the student can add the numbers.

Casually write the answer on the blackboard.

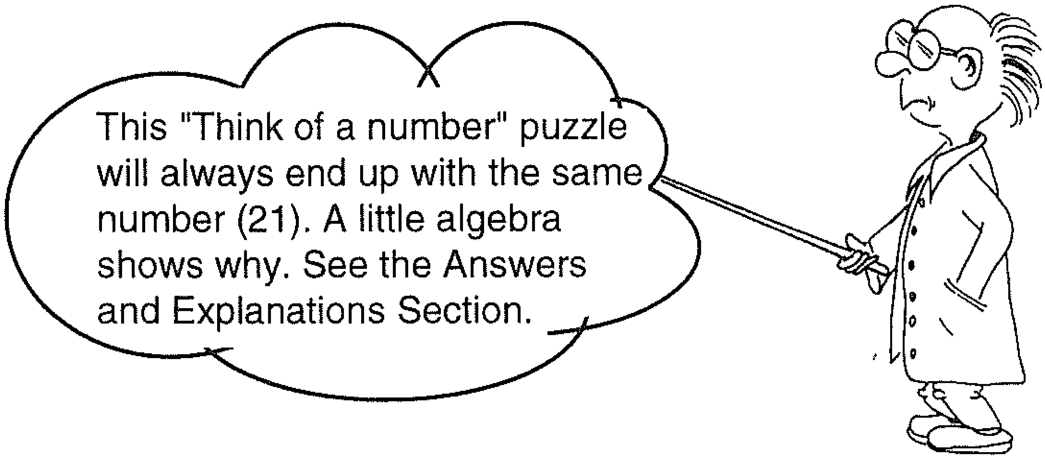
THINK OF A NUMBER I

➠ Ask your student(s) to think of a number.	**e.g. 20**
➠ Treble it (Multiply by 3).	**60**
➠ Add ten.	**70**
➠ Double it.	**140**
➠ Subtract 14.	**126**
➠ Divide by 6.	**21**
➠ Subtract 1.	**?**



THINK OF A NUMBER II

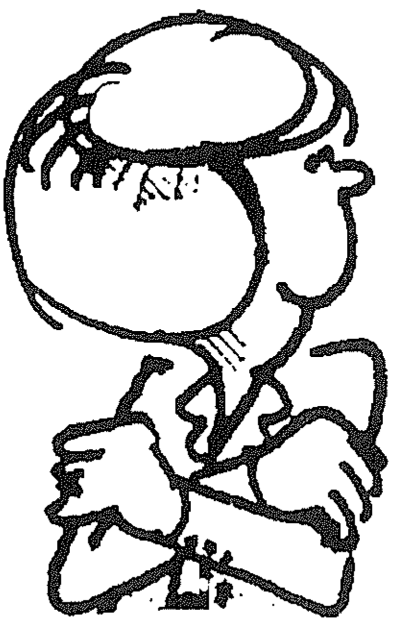
➠ Ask your student(s) to think of a number.	**e.g. 15**
➠ Double it.	**30**
➠ Add one hundred.	**130**
➠ Halve it.	**65**
➠ Subtract 29.	**36**
➠ Take away the number you first thought of.	**?**



SHOWING YOUR AGE

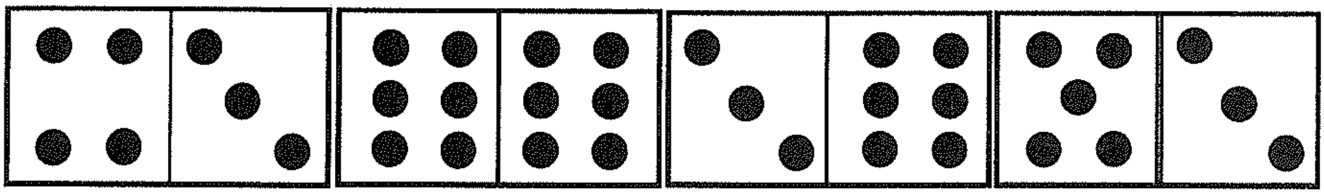
To find a person's age and date of birth pass them a calculator with the display covered and then ask them to follow these instructions.

- Enter the month in which you were born. (January = 1, February = 2, etc.) followed by the last two digits of the year in which you were born.
- Multiply by 2, then by 5 and finally 10.
- Add your original number (formed by entering your birth month followed by your birth year) to the total.
- Add 24 for the number of hours in a day.
- Add 7 for the number of days in a week.
- Double 52, for the number of weeks in a year and add it to your total.
- Finally add 365 for the number of days in a year.
- Ask the participant to pass the calculator to you. Uncover the display and offer to reveal their age and the month and year in which they were born.

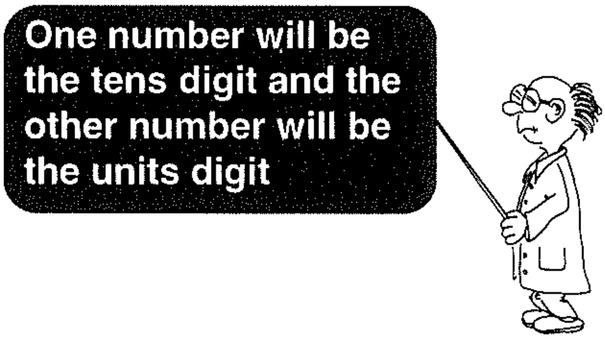


To discover the person's age, look at the last two digits and compare it to the current year. If a five figure digit number is formed, then the digit in the ten thousnd's place represents the month in which they were born (i.e. Jan = 1, Feb = 2) and the last two digits represent the year. Disregard the middle digits.
If a six figure number is formed, it means the person was born in October, November or December and the first two digits represent the month. Disregard the two middle digits and the last two digits represent the year.

DIABOLIC DOMINOES



- Ask a student to choose any domino piece, and then follow this procedure.
- Multiply one of the numbers shown on the domino by **5**.
- Add **6**.
- Double your answer.
- Add the number on the other half of the domino.
- Ask the student to tell you their final number.
- Subtract **12** to find the two numbers that were on the original domino.



Variations

Use two dice instead of a domino. Make up a different set of instructions.

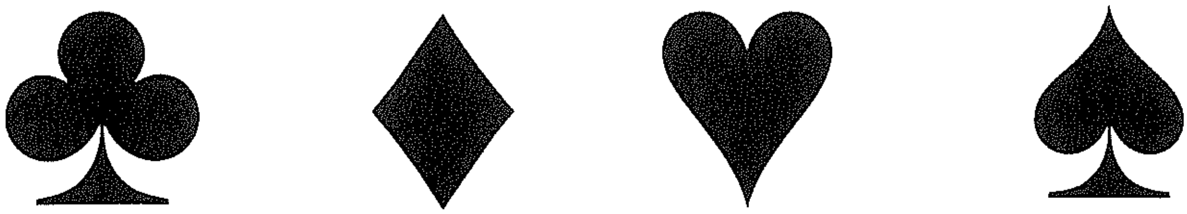
e.g. Multiply one of the numbers by **5.**
Add **7.**
Double it.
Add the number shown on the other die.
Subtract **14** to find the numbers.

PICK A CARD – ANY CARD

You will need a deck of cards.

Ask your volunteer to

❤ Pick a card and keep it secret.	**e.g. 9 of clubs**
❤ Double the face value.*	**18**
❤ Add **4.**	**22**
❤ Multiply by **5.**	**110**
❤ Add the suit value (in this case **2** – see below).	**112**

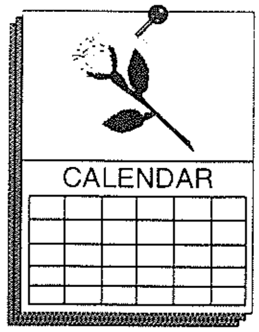


2 for a club　4 for a diamond　6 for a heart　8 for a spade

❤ Next ask your volunteer to tell you his or her final number and then offer to tell which card he or she chose.

To identify the card subtract 20 from the result. The tens digit gives the face value of the card and the units digit gives the suit.
e.g. 112 – 20 = 92 or 9 of clubs

MAGIC MONTHS



- Ask a member of the class to choose a 3 x 3 block from any month on the calendar, but to make sure that you do not see it.

2001

JANUARY

S	M	T	W	T	F	S
	1	2	3	4	5	6
7	8	9	10	11	12	13
14	15	16	17	18	19	20
21	22	23	24	25	26	27
28	29	30	31			

- Next ask the student to add the nine numbers.
  e. g. 11 + 12 + 13 + 18 + 19 + 20 + 25 + 26 + 27 = 171

- Ask the student to tell you the total.

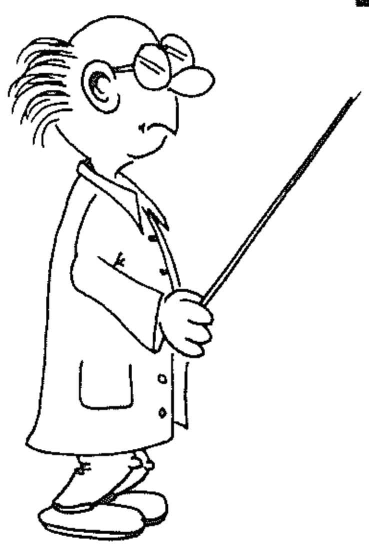
- Mentally divide the total by 9, which will reveal the middle number in the block. $171 \div 9 = 19$

- Announce to the class that you will call out the original nine numbers which were chosen.

Subtract 8 to determine the first number in the block. The rest may be worked out using the sequence of the calendar.

DIVISION DECISION II

- Ask a volunteer to write any three digit number on the blackboard. **731**
- Check whether the number is divisible by **9** using a calculator. **NO**
- Offer to add a digit anywhere in the three digit number to produce a four digit number that is divisible by **9.**



To make this Mathematical Novelty work all that you need to do is add the digits together and subtract from the next multiple of nine

i.e. 7 + 3 + 1 = 11,
18 – 11 = 7

The digit that is found may be added anywhere in the number.

e.g. 7731, 7371 or 7317

Variation

The size of the original number does not matter, although it is probably a good idea to restrict the number to seven digits or less to enable the result to be checked on a calculator.

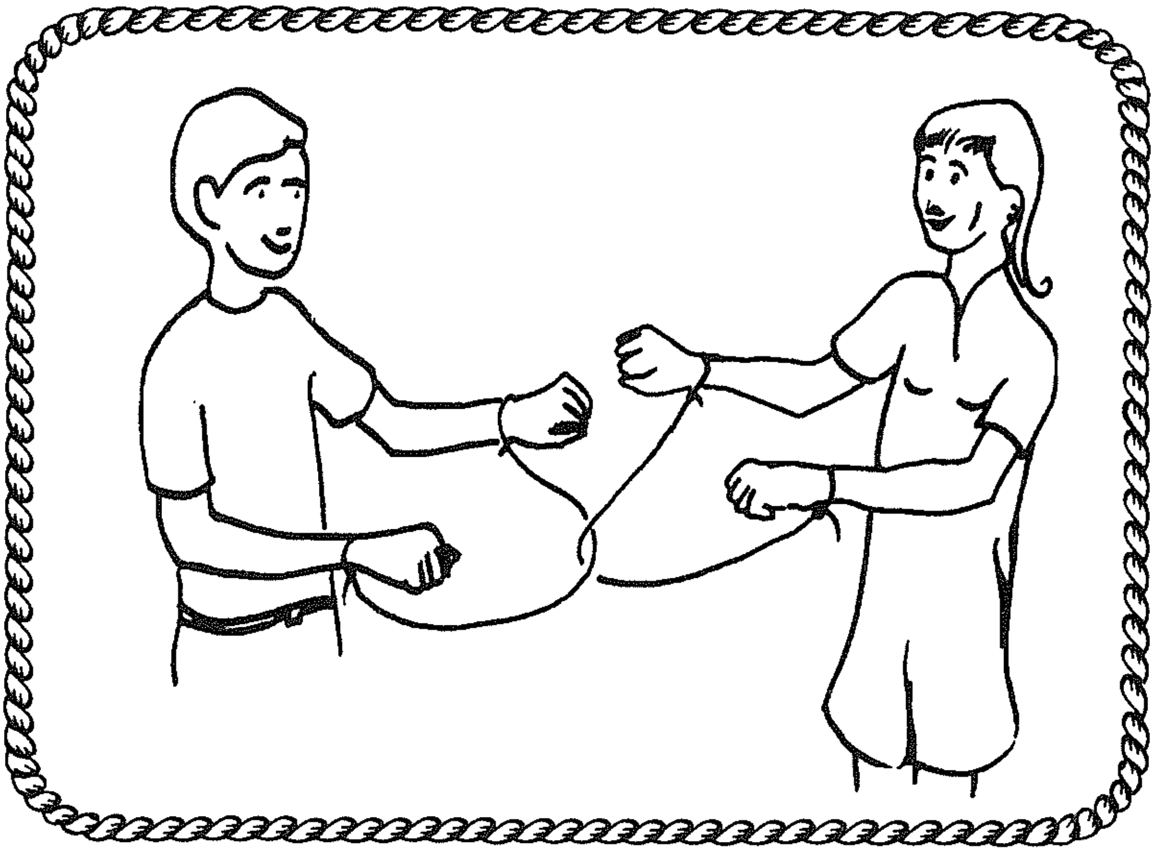
A similar procedure may be used to create a number that is divisible by three. To create a number divisible by three add all the digits and subtract from the next multiple of three to find a suitable digit.

TIED UP IN KNOTS

To perform the following trick you will need two pieces of string each about 1 metre in length.

- Ask for two volunteers.
- Loosely tie one piece of string onto each wrist of the first volunteer.
- Loop the second piece of string over the first piece and then tie it to the wrists of a second volunteer.

The trick is for the two students to separate themselves without cutting or untying the string.



NUMBER STRIPS

To perform the following trick you will need to prepare four cardboard strips with the following numbers printed on the front and back. See the Answers and explanations section for instructions on how to create your own number strips.

Front

7	3	2	9
6	4	1	8
3	8	7	3
9	4	6	6
1	5	5	2

Back

8	5	3	6
3	0	7	2
4	1	6	8
2	5	9	3
6	9	2	3

Give a student the four strips and ask him/her to arrange them to form five four digit numbers

e.g

7	5	2	6
6	0	1	2
3	1	7	8
9	5	6	3
1	9	5	3

Challenge the students to add the five numbers before you do.

While they are busy adding the five numbers simply add 22 220 to the second four digit number to find the answer.

e.g. 22 220
+ 6 012
= 28 232

MIND READER

Ask your student(s) to follow these steps.

- Choose a four digit number where all the digits are different. **7351**
- Reverse the order of the digits. **1537**
- Take the smaller number away from the larger number.

$$\begin{array}{r} 7351 \\ -\underline{1537} \\ =\underline{5814} \end{array}$$

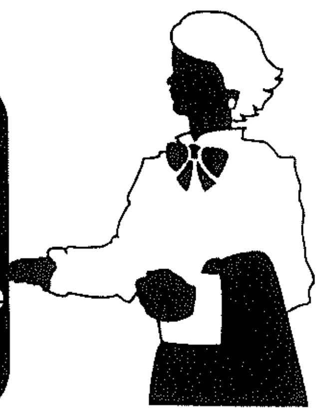
- Multiply your answer by any number from **2** to **9999**

$$\begin{array}{r} 5814 \\ \underline{\times 3} \\ =\underline{17\,442} \end{array}$$

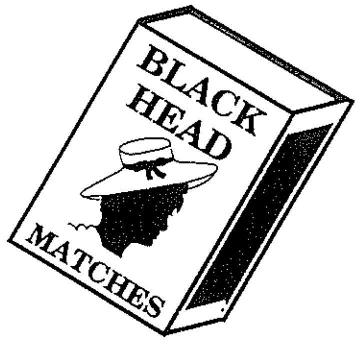
- Cross off any digit from your answer except **0** or **9** **1~~7~~ 442**
- Now ask your student to tell you what his/her remaining digits are. Note, the order is not important.
- Offer to "read his/her mind" to find out the number which was crossed off.

To work out which digit was crossed off simply add all the digits and subtract your answer from the next multiple of 9

Using the above example, the digits 1 + 4 + 4 + 2 = 11. The next multiple of 9 is 18, therefore the missing digit is 7.



MYSTERY MATCHBOX



- Ask a volunteer to secretly count the number of matches in a matchbox. **e.g. 48**

- Next ask the volunteer to add the two digits of the number. $4 + 8 = 12$

- Instruct the volunteer to remove this number of matches from the box and close the matchbox

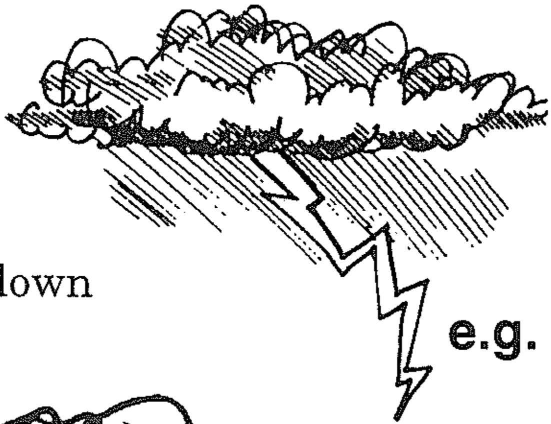
$$\begin{array}{r} 48 \\ -12 \\ \hline =36 \\ \hline \end{array}$$

- Ask the volunteer to hand the matchbox to you. Shake and then announce the number of matches contained in the box.

If the volunteer has correctly carried out the above steps then a multiple of nine is always left. All you have to do is to guess from the feel of the box whether it contains 45, 36, 27, 18 or 9 matches.

To eliminate the possibility that you may have rigged the box you could get the volunteer to secretly remove some matches from the box, before starting to count them.

LIGHTNING ADDITION



Ask your students to write down any two numbers. **e.g. 2, 6**

The first time you try this Mathematical Novelty it is a good idea to start with two small numbers

The idea is to create a sequence of ten numbers using the chosen numbers as the starting point.

To create the third number in the sequence add the first two numbers. **2 + 6 = 8**

To create the fourth number in the sequence add the second and third number. **6 + 8 = 14**

Continue this way until you reach the tenth number in the sequence.

2, 6, 8, 14, 22, 36, 58, 94, 152, 246.

↑ seventh number

Ask your students to show you the set of numbers for a short period of time (a few seconds is enough). Note the seventh number in the sequence.

Instruct the students to add the ten numbers.

While the students are performing the addition mentally multiply the seventh number in the sequence by eleven and casually write the number on the board. **11 x 58 = 638**

See the Answers and Explanations section for a shortcut method of multiplying by eleven.

SUPER SEQUENCES

Ask a volunteer to write down any number they like.

e.g. 10

When trying this Mathematical Novelty for the first time it would be a good idea to restrict the chosen number to less than twenty. If larger numbers are allowed you will probably need a calculator.

- Next ask the volunteer to add all the numbers up to and including the chosen number.

1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10 = 55

- While the class is hard at work adding the numbers casually write the answer on the blackboard.

To work out the answer quickly multiply the last number in the sequence by one more than the number and then halve the result.

e.g. 10 x 11 = 110 ÷ 2 = 55

Variation

A variation on the above sequence involves adding all the odd numbers up to a particular number.

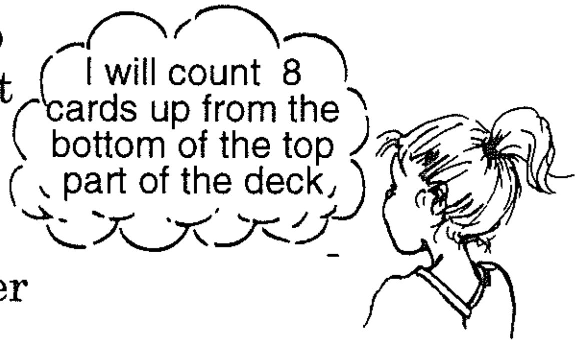
e.g. 1 + 3 + 5 + 7 + 9 + 11 + 13 + 15...

Try using a sequence of consecutive even numbers.

e.g. 2 + 4 + 6 + 8 + 10

ALL MATHS IS WONDERFUL

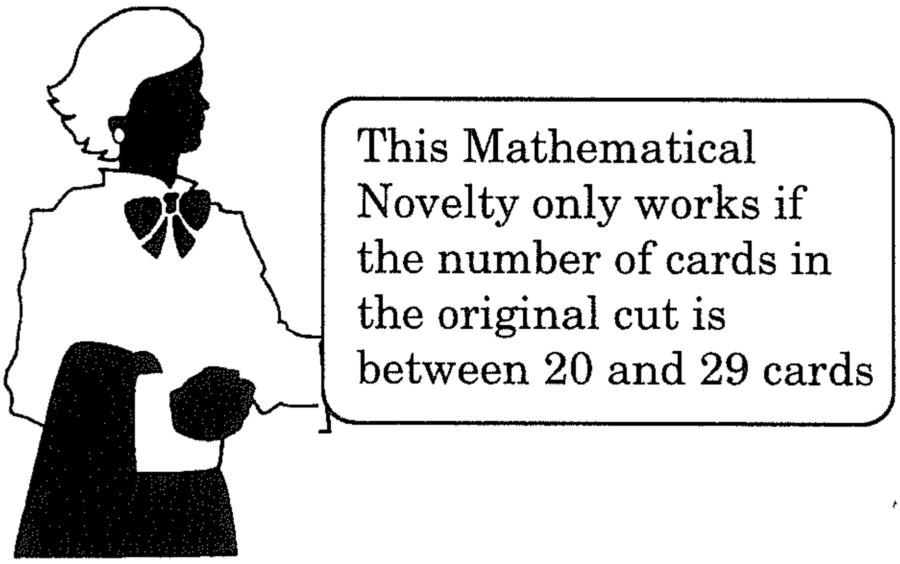
- Cut a deck of cards close to the middle.
- Count the cards in the top part of the cut **26**
- Add the digits together. **2 + 6 = 8**
- Count this number of cards up from the bottom of the top part of the deck to reveal the magic card.



- Now put the deck back together in its original order.
- The magic card may be found by spelling out the phrase

A L L M A T H S I S W O N D E R F U L.

and dealing out the cards one at a time as each letter is spoken. The magic card will be the final card.(L)



ANSWERS AND EXPLANATIONS

Page 5. Ten Times

This novelty may easily be altered by changing the second instruction "Multiply by 10 " to "add a zero to the end". Both instructions have the same effect – moving the number one decimal place to the left.

Why it works.

Let the random number be "a".
Multiplying by 10 gives $10a$.
Subtracting the original number $10a - a = 9a$.
Dividing by the original number, $9a \div a$, leaves 9.

Page 6. Give me Five.

Let the number chosen be n.
"Adding the next number" gives $n + (n + 1)$ or $2n + 1$.
"Adding 9" gives $2n + 10$.
Dividing by 2 gives $n + 5$,
and subtracting the original number (n) leaves 5.
The Novelty may easily be altered by adding a different odd number instead of 9 .
The answer will always be half of one more than the odd number added.

Page 7. Role Reversal

If we call our starting digits x and y, then the two digit number would be $10x + y$
Reversing the digits we get $10y + x$

$$(10x + y) - (10y + x) \text{ leaves } 9x - 9y \quad \text{or} \quad 9(x - y)$$

This means that we will always end up with a multiple of 9 after the first subtraction. Repeating the process eventually leads us to the point where the difference between the digits x and y is 1 and $9(x - y) = 9$

Page 8. CYCLING DIGITS

If we let the four digits be represented by a, b, c and d then cycling the digits gives

$$\begin{aligned}
&(1000a + 100b + 10c + d) \\
&+ (1000d + 100a + 10b + c) \\
&+ (1000c + 100d + 10a + b) \\
&+ (1000b + 100c + 10d + a) \\
&= (1111a + 1111b + 1111c + 1111d \text{ or } 1111(a + b + c + d)
\end{aligned}$$

Dividing by the sum of all the digits ($a + b + c + d$) leaves 1111
If you begin with a 3 digit number the result will always be 111, and if you begin with a 5 digit number the result will always be 11 111.
Starting with 4 digits the same or any number of digits the same will not make any difference to the result.

Page 9. DOUBLE UP

This Mathematical Novelty is the opposite of Tautonyms contained in Number Novelties.
Multiplying the original 3 digit number by 7, 11 and 13 is really the same as multiplying by 1001. Multiplying by the 1000 moves the three digit number 3 decimal places to the left, while multiplying by 1 ensures that the same digits will be repeated in the hundreds, tens and units places.

The order in which the multiplication is carried out will make no difference to the final answer.
There are several variations to this novelty.
For example you can reduce the number of steps by asking your students to multiply by 7 and then by 143 (11 x 13) or by 11 and 91 (7 x 13)
Another variation involves starting with a two digit number and multiplying by 13, 21 and 37. Effectively this is the same as multiplying by 10 101 and therefore the same pair of digits is repeated twice. e.g. 64 x 13 x 21 x 37 = 646464. This novelty may also be varied by changing the order in which the multiplication occurs.
Both novelties may be altered to incorporate division rather than multiplication. A detailed explanation is contained in Number Novelties.

Page 10. DO OR DIE

The answer is always 49. A little Algebra helps to show why.
If we call the numbers on the top of each die a and b then the numbers on the bottom of each die will be $7 - a$ and $7 - b$, because the numbers on opposite faces of a die always add up to 7.
Multiplying the top numbers $a \times b$ we get ab.
Multiplying the bottom numbers $(7 - a)(7 - b)$ we get $49 - 7a - 7b + ab$.
(A knowledge of how to multiply binomials is required to understand this part. This trick could be used to develop the topic of multiplying binomials).
Multiplying the top number from the first die by the bottom number from the second die $a(7 - b)$ gives $7a - ab$.
Multiplying the top number from the second die by the bottom number from the first die $b(7 - a)$ gives $7b - ab$.
Combining the results we have
$(ab) + (49 - 7a - 7b + ab) + (7a - ab) + (7b - ab)$
which when simplified leaves 49.

Page 11. DOUBLE TROUBLE

This Mathematical Novelty is similar to Seeing Double contained in Number Novelties.
Let a, b and c represent the single digit numbers. The nine two digit numbers formed are:

$10a + a$	$10a + b$	$10a + c$
$10b + a$	$10b + b$	$10b + c$
$10c + a$	$10c + b$	$10c + c$

Adding the terms we get $33a + 33b + 33c = 33(a + b + c)$
Dividing by $(a + b + c)$, the original three digits gives 33 every time.

Page 12. THIRTY SEVEN

3 x 37 = 111	4 x 37 = 148	5 x 37 = 185
6 x 37 = 222	7 x 37 = 259	8 x 37 = 296
9 x 37 = 333	10 x 37 = 370	11 x 37 = 407
12 x 37 = 444	13 x 37 = 481	14 x 37 = 518

15 x 37 = 555, 18 x 37 = 666, 21 x 37 = 777, 24 x 37 = 888

This pattern occurs because 3 x 37 = 111.
No remainders.
In every case numbers are formed which are evenly divisible by 37. This is also the case if the three arrangements are added first.

Page 13. WHAT DAY WAS IT?

This procedure may be used to determine on what day of the week any date falls, or finding the day on which certain famous events occurred. e.g. Walking on the Moon. To verify your result you may like to use the Perpetual Calendar.
The following Perpetual Calendar first appeared in Mathematics Teaching 143, June 1993, p. 33 and was devised by S. Vaithianathan, Epsom, Surrey. A few minor changes have been made.
The chart shown below may be used to find the day of the week on which a particular date falls. Three pieces of information are required.

		Ref. No.
• The Month	e.g. January	5
• The first two digits of the year	19	2
• The last two digits of the year	81	4
	TOTAL	11

The reference numbers are found by consulting the chart below.

Ref. No.	0	1	2	3	4	5	6
Months	Aug, *Feb**	Feb, Mar, Nov	June	Sep, Dec	Apr, Jul, *Jan**	Jan, Oct	May
Years (first two digits) Julian Calendar	06 13	05 12	04 11	03 10 17	02 09 16	01 08 15	00 07 14
Gregorian Calendar		16 20 24	15 19 23		18 22		17 21 25
(Last two digits)	05 11 16 22 33 39 44 50 61 67 72 78 89 95	00 06 17 23 28 34 45 51 56 62 73 79 84 90	01 07 12 18 29 35 40 46 57 63 68 74 85 91 96	02 13 19 24 30 41 47 52 58 69 75 80 86 87	03 08 14 25 31 36 42 53 59 64 70 81 87 92 98	09 15 20 26 37 43 48 54 65 71 76 82 93 99	04 10 21 27 32 38 49 55 60 66 77 83 88 94

Gregorian Calendar: Adopted in Catholic Countries in 1582 and in England and colonies from 1752 on.

* Jan and Feb in italics for leap years only. A leap year is held every fourth year. Century years do not count as leap years unless they are evenly divisible by 400.

The reference numbers are then added and the appropriate calendar consulted. For example calendar 11 gives the days and dates for January 1981.

0, 7, 14

Sun	Mon	Tue	Wed	Thur	Fri	Sat
1	2	3	4	5	6	7
8	9	10	11	12	13	14
15	16	17	18	19	20	21
22	23	24	25	26	27	28
29	30	31				

1, 8, 15

Sun	Mon	Tue	Wed	Thur	Fri	Sat
	1	2	3	4	5	6
7	8	9	10	11	12	13
14	15	16	17	18	19	20
21	22	23	24	25	26	27
28	29	30	31			

2, 9, 16

Sun	Mon	Tue	Wed	Thur	Fri	Sat
		1	2	3	4	5
6	7	8	9	10	11	12
13	14	15	16	17	18	19
20	21	22	23	24	25	26
27	28	29	30	31		

3. 10. 17

Sun	Mon	Tue	Wed	Thur	Fri	Sat
			1	2	3	4
5	6	7	8	9	10	11
12	13	14	15	16	17	18
19	20	21	22	23	24	25
26	27	28	29	30	31	

4, 11, 18

Sun	Mon	Tue	Wed	Thur	Fri	Sat
				1	2	3
4	5	6	7	8	9	10
11	12	13	14	15	16	17
18	19	20	21	22	23	24
25	26	27	28	29	30	31

5,12

Sun	Mon	Tue	Wed	Thur	Fri	Sat
					1	2
3	4	5	6	7	8	9
10	11	12	13	14	15	16
17	18	19	20	21	22	23
24	25	26	27	28	29	30
31						

6, 13

Sun	Mon	Tue	Wed	Thur	Fri	Sat
						1
2	3	4	5	6	7	8
9	10	11	12	13	14	15
16	17	18	19	20	21	22
23	24	25	26	27	28	29
30	31					

Page 14. CALENDAR MAGIC

Adding eight to the smallest number in the box and multiplying by nine gives the same result as adding the nine dates in the box.
This procedure will work for any month and any year.
We can see why it works if we use 'd' to represent the smallest date in the set of nine dates.

d	$d + 1$	$d + 2$
$d + 7$	$d + 8$	$d + 9$
$d + 14$	$d + 15$	$d + 16$

Adding eight to the smallest number gives $d + 8$.
Multiplying by 9 gives $9(d + 8)$ or $9d + 72$.
Adding the nine dates gives:
$d + (d + 1) + (d + 2) + (d + 7) + (d + 8) + (d + 9) + (d + 14) + (d + 15) + (d + 16)$
or $9d + 72$

Page 15. COOL CALENDAR

Consider the four dates

d	$d + 1$
$d + 7$	$d + 8$

Adding the four dates $d + (d + 1) + (d + 7) + (d + 8)$ gives $4d + 16$
Dividing by 4 gives $d + 4$
Subtracting 4 leaves d or the date in the top left corner of the box.

Variations

This Calendar Novelty may be altered in several ways. Here are some suggestions. Instead of subtracting 4, you might subtract 3, which would produce a result of $d + 1$ or the date in the top right hand corner of the box. Adding 3 would produce $d + 7$ or the date in the bottom left hand corner of the box, whilst adding 4 would produce $d + 8$ or the date in the bottom right hand corner of the box.

Page 16. CALENDAR CAPER

This Mathematical Novelty is similar to the Tricky Table Number Novelty contained in the book "Number Novelties"
The students should find that their circled numbers will always add to the same value. This occurs because of the uniform nature of the Calendar. See "Number Novelties" for a more detailed explanation of why it works.

Page 17. CRAZY CALENDAR

If the 16 dates are denoted by:

d	$d + 1$	$d + 2$	$d + 3$
$d + 7$	$d + 8$	$d + 9$	$d + 10$
$d + 14$	$d + 15$	$d + 16$	$d + 17$
$d + 21$	$d + 22$	$d + 23$	$d + 24$

then the total arrived at by adding the numbers in the four corners is:
$d + (d + 3) + (d + 21) + (d + 24)$ or $4d + 48$.
Choosing 4 numbers from different rows and columns ensures that a combination adding to $4d + 48$ occurs.
This Mathematical Novelty may be altered into a Mind Reading trick by changing the instructions. i.e. Ask a volunteer to draw a 4 x 4 box around any 16 dates and then show you. Carefully note a pair of dates contained in diagonally opposite corners of the box. Add these two dates and then double the total. Remember this total.
Ask the volunteer to circle 4 numbers using the instructions from Crazy Calendar Mathematical Novelty. This should be done out of your sight. Once completed announce the total to the volunteer.

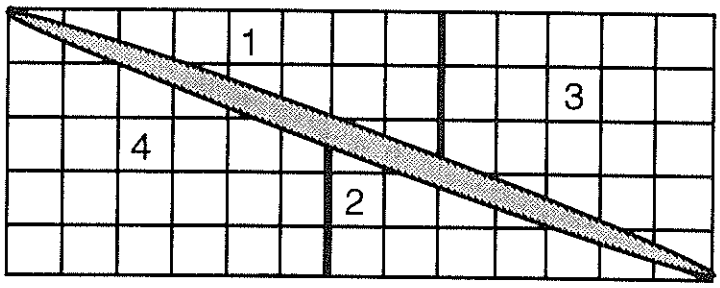
Page 18. NUMBER WORDS

44	72	77	23
Forty Four	Seventy two	Seventy Seven	Twenty three
9	10	12	11
Nine	Three	Twelve	Eleven
4	5	6	6
	Five	Six	Six
	4	3	3
		Three	Three
		5	5
		Five	Five
		4	4

Once you reach 4 an endless loop forms because four is made up of 4 letters.

Page 19. EXPANDING AREA

The square has an area of 64 square units, while it would appear that the rectangle has an area of 65 square units (13 x 5). The reason for the apparent discrepancy is that the pieces along the diagonal of the rectangle do not quite fit together. There is a gap in the shape of a parallelogram which has an area of 1 square unit, which makes up the difference.



Page 20. A SHAPE WITH A TWIST I

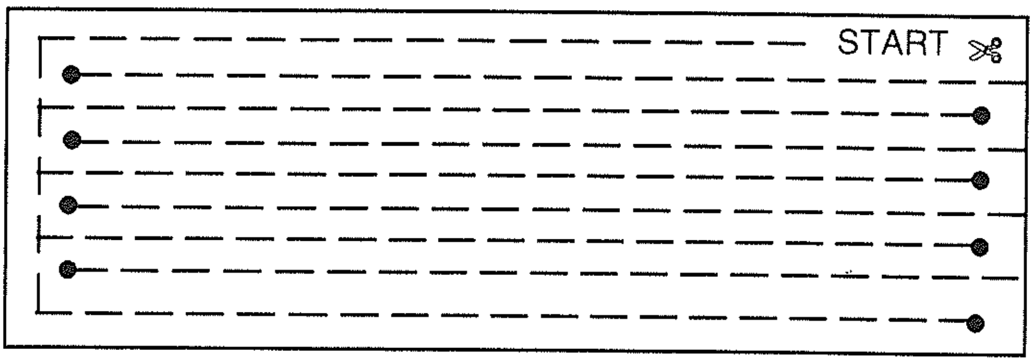
The Moebius band formed by a half twist has only one side.

Page 21. A SHAPE WITH A TWIST II

When a Moebius with a half twist is cut down the centre one large loop is formed.

Page 22 . STEPPING THROUGH PAPER

If you wish to make the effect more dramatic you can start with a much smaller piece of paper and still walk through it.
There are several ways that a piece of paper may be cut in order to walk through it.
e.g.
Draw lines on your sheet of paper and cut along these lines. Start cutting the **outside** lines first. Stop cutting at each of the dots.



Note, by cutting at smaller intervals a larger opening is created.

Page 23. CALCULATION SHORTCUTS

SQUARING ANY TWO DIGIT NUMBER

The procedure for squaring a two digit number may be shown algebraically.
If 'a' represents the number to be squared and 'b' represents the unit part of the number then $a^2 = (a - b)(a + b) + b^2$

(a) $23^2 = (20 \times 26) + 3^2 = 520 + 9 = 529$

(b) $32^2 = (30 \times 34) + 2^2 = 1020 + 4 = 1024$

(c) $35^2 = (30 \times 40) + 5^2 = 1200 + 25 = 1225$

(d) $41^2 = (40 \times 42) + 1^2 = 1680 + 1 = 1681$

(e) $43^2 = (40 \times 46) + 3^2 = 1840 + 9 = 1849$

(f) $49^2 = (50 \times 48) + 1^2 = 2400 + 1 = 2401$

(g) $52^2 = (50 \times 54) + 2^2 = 2700 + 4 = 2704$

(h) $67^2 = (70 \times 64) + 3^2 = 4480 + 9 = 4489$

(i) $94^2 = (90 \times 98) + 4^2 = 8820 + 16 = 8836$

Page 24. CALCULATION SHORTCUTS

MULTIPLYING RELATED NUMBERS

This shortcut is based on the difference of squares

i.e. $(a - b)(a + b) = a^2 - b^2$

(a) $33 \times 27 = (30 + 3)(30 - 3) = 900 - 9 = 891$

(b) $31 \times 29 = (30 + 1)(30 - 1) = 900 - 1 = 899$

(c) $44 \times 36 = (40 + 4)(40 - 4) = 1600 - 16 = 1584$

(d) $42 \times 38 = (40 + 2)(40 - 2) = 1600 - 4 = 1596$

(e) $51 \times 49 = (50 + 1)(50 - 1) = 2500 - 1 = 2499$

(f) $53 \times 47 = (50 + 3)(50 - 3) = 2500 - 9 = 2491$

(g) $68 \times 72 = (70 - 2)(70 + 2) = 4900 - 4 = 4896$

(h) $398 \times 402 = (400 - 2)(400 + 2) = 160\,000 - 4 = 159\,996$

(i) $797 \times 803 = (800 - 3)(800 + 3) = 640\,000 - 9 = 639\,991$

Page 25. CALCULATION SHORTCUTS

DIVIDING BY 5

(a) $275 \div 5 \rightarrow 550 \rightarrow 55$

(b) $414 \div 5 \rightarrow 828 \rightarrow 82{\cdot}8$

(c) $665 \div 5 \rightarrow 1330 \rightarrow 133$

(d) $491 \div 5 \rightarrow 982 \rightarrow 98{\cdot}2$

(e) $304 \div 5 \rightarrow 608 \rightarrow 60{\cdot}8$

MULTIPLYING BY 15

(a) $342 \times 15 \rightarrow 3420 + 1710 = 5130$

(b) $416 \times 15 \rightarrow 4160 + 2080 = 6240$

(c) $548 \times 15 \rightarrow 5480 + 2740 = 8220$

(d) $643 \times 15 \rightarrow 6430 + 3215 = 9645$

(e) $781 \times 15 \rightarrow 7810 + 3905$, $11\,715$

Page 26. CASTING OUT NINES

Casting out nines was used hundreds of years ago to check calculations. Casting out nines depends on the fact that adding the digits of a number and repeating until a single digit remains gives the remainder when the number is divided by 9. If the remainder is 9 the remainder is in fact zero. Teachers should note that while "casting out nines" is an efficient method of checking answers it does not guarantee that the answer is correct. It does however give a fairly good indication that the answer is probably correct. In the case of division it only checks the remainder.

(a)	657 → 6 + 5 + 7 → 18 → 1 + 8 → **9**	No remainder
(b)	848 → 8 + 4 + 8 → 20 → 2 + 0 → **2**	Remainder = 2
(c)	514 → 5 + 1 + 4 → 10 → 1 + 0 → **1**	Remainder = 1
(d)	748 → 7 + 4 + 8 → 19 → 1 + 9 → 10 → 1 + 0 → **1**	Remainder = 1
(e)	432 → 4 + 3 + 2 → **9**	No remainder
(f)	329 → 3 + 2 + 9 → 14 → 1 + 4 → **5**	Remainder = 5
(g)	666 → 6 + 6 + 6 → 18 → 1 + 8 → **9**	No Remainder
(h)	1018 → 1 + 0 + 1 + 8 → 10 → 1 + 0 → **1**	Remainder = 1

The remainder may be found by "casting out nines."

Page 28. CALCULATION CHECKS +

(a)
527 → 5 }
+ 258 → 6 } 5 + 6 → 11 → **2**
= 775 → **1**

The two do not match therefore the answer is wrong — should be 785

(b)
144 → 9 }
+ 717 → 6 } 9 + 6 → 15 → **6**
= 861 → 15 → **6** The two match so the answer is probably correct.

(c)
462 → 3 }
+ 349 → 7 } 3 + 7 → 10 → **1**
= 801 → **9**

The two do not match therefore the answer is wrong — should be 811.

(d)
371 → 2 }
+ 243 → 9 } 11 → **2**
= 614 → 11 → **2** The two match so the answer is probably correct.

(e)
576 → 9 }
+ 248 → 5 } 14 → **5**
= 914 → **5**

Here is an example where the "casting out nines" method fails. Even though the values match the answer is wrong — should be 824.

You will note that the digit values for 914 and 824 are exactly the same. Teaching point: The children should be made aware that while the "casting out nines" method works well most of the time, it is not foolproof.

(f)
256 → 4 }
+ 732 → 3 } 4 + 3 → **7**
= 888 → **6**

The two values do not match therefore the answer is wrong — should be 988.

Page 30 . CALCULATION CHECKS —

(a)
$$\begin{array}{rcll} 517 & \rightarrow & \textcircled{4} & \\ -76 & \rightarrow & 4 & \multirow{2}{*}{$\}$ 4 + 9 \rightarrow 13 \rightarrow \textcircled{4}} \\ =441 & \rightarrow & 9 & \end{array}$$

The two values match so the answer is probably correct.

(b)
$$\begin{array}{rcll} 676 & \rightarrow & \textcircled{1} & \\ -368 & \rightarrow & 8 & \\ =318 & \rightarrow & 3 & \} \; 8 + 3 \rightarrow 11 \rightarrow \textcircled{2} \end{array}$$

The two values do not match therefore the answer is wrong — should be 308.

(c)
$$\begin{array}{rcll} 873 & \rightarrow & \textcircled{9} & \\ -491 & \rightarrow & 5 & \\ =372 & \rightarrow & 3 & \} \; 5 + 3 \rightarrow \textcircled{8} \end{array}$$

The two values do not match therefore the answer is wrong — should be 382.

(d)
$$\begin{array}{rcll} 2417 & \rightarrow & \textcircled{5} & \\ -936 & \rightarrow & 9 & \\ =1481 & \rightarrow & 5 & \} \; 9 + 5 \rightarrow 14 \rightarrow 1 + 4 \rightarrow \textcircled{5} \end{array}$$

The two values match therefore the answer is probably correct.

(e)
$$\begin{array}{rcll} 4635 & \rightarrow & \textcircled{9} & \\ -2998 & \rightarrow & 1 & \\ =1647 & \rightarrow & 9 & \} \; 1 + 9 \rightarrow 10 \rightarrow \textcircled{1} \end{array}$$

The two values do not match therefore the answer is wrong — should be 1637.

(f)
$$\begin{array}{rcll} 1078 & \rightarrow & \textcircled{7} & \\ -959 & \rightarrow & 5 & \\ =109 & \rightarrow & 1 & \} \; 5 + 1 \rightarrow \textcircled{6} \end{array}$$

The two values do not match therefore the answer is wrong — should be 119.

Page 31. CALCULATION CHECKS X

(a)
$$\begin{array}{rcccll} 37 & \rightarrow & 10 & \rightarrow & 1 & \\ \times 16 & \rightarrow & 7 & & & \} \; 1 \times 7 \rightarrow \textcircled{7} \\ =592 & \rightarrow & 16 & \rightarrow & 1 + 6 = \textcircled{7} & \end{array}$$

The two values match therefore the answer is probably correct.

(b)
$$\begin{array}{rcccll} 54 & \rightarrow & 5 + 4 & \rightarrow & 9 & \\ \times 36 & \rightarrow & 3 + 6 & \rightarrow & 9 & \} \; 9 \times 9 \rightarrow 81 \rightarrow 8 + 1 \rightarrow 9 \\ =1934 & \rightarrow & 1 + 9 + 3 + 4 & \rightarrow & 17 & \rightarrow 1 + 7 \rightarrow 8 \end{array}$$

The two values do not match therefore the answer is wrong — should be 1944.

(c) 91 → 9 + 1 → 10 → 1 } 1 x 8 → 8
x 62 → 6 + 2 → 8
= 5662 5 + 6 + 6 + 2 → 19 → 1 + 9 → 10 → 1

The two values do not match therefore the answer is wrong — should be 5642.

(d) 67 → 6 + 7 → 13 → 1 + 3 → 4 } 4 x 1 → 4
x 73 → 7 + 3 → 10 → 1
= 4891 → 4 + 8 + 9 + 1 → 22 → 2 + 2 → 4

The two values match therefore the answer is probably correct.

(e) 83 → 8 + 3 → 11 → 1 + 1 → 2 } 2 x 4 → 8
x 67 → 6 + 7 → 13 → 1 + 3 → 4
= 5561 → 5 + 5 + 6 + 1 → 17 → 1 + 7 → 8

The two values match therefore the answer is probably correct.

(f) 87 → 8 + 7 → 15 → 1 + 5 → 6 } 6 x 7 → 42 → 4 + 2 → 6
x 34 → 3 + 4 → 7
= 2858 → 2 + 8 + 5 + 8 → 23 → 2 + 3 → 5

The two values do not match therefore the answer is wrong — should be 2958.

Page 32. CALCULATION CHECKS ÷

(a) 38 → 3 + 8 → 11 → 1 + 1 → 2
7)266 Divisor → 7
Multiply the two values 2 x 7 14
Cast out nines 14 → 1 + 4 → **5**
Cast out nines for dividend 266 → 2 + 6 + 6 → 14 → 1 + 4 → **5**
The two values match therefore the answer is probably correct.

(b) 66 → 6 + 6 → 12 → 1 + 2 → 3
8)536 Divisor → 8
Multiply the two values 3 x 8 → 24
Cast out nines 24 → 2 + 4 → **6**
Cast out nines for dividend 536 → 5 + 3 + 6 → 14 → 1 + 4 → **5**
The two values do not match therefore the answer is wrong.
— should be 67.

(c) 84 r 3 → 8 + 4 → 12 → 1 + 2 → 3
6)506 Divisor → 6
Multiply the two values 3 x 6 → 18
Add remainder + 3
Total 21
Cast out nines 21 → 2 + 1 → **3**
Cast out nines for dividend 506 → 5 + 0 + 6 → 11 → 1 + 1 → 2
The two values do not match therefore the answer is wrong
— should be 84 r 2.

(d) 93 → 9 + 3 → 12 1 + 2 → 3
12)1116 Divisor → 1 + 2 → 3
Multiply the two values 3 x 3 → **9**
No need to cast out nines in this case
Cast out for dividend 1116 → 1 + 1+ 1+ 1 + 6 → **9**
The two values match therefore the answer is probably correct.

(e)	$13\overline{)728}$ 59	→	5 + 9	→	14	→	1 + 4	→	5	
			Divisor		→		1 + 3	→	4	
			Multiply the two values				5 x 4	→	20	
			Cast out nines 20					→	**2**	
Cast out for dividend			728	→	7 + 2 +8	→	17	→	1 + 7	→ **8**

The two values do not match therefore the answer is wrong – should be 56.

(f)	$14\overline{)1096}$ 77 r 12	→	7 + 7	→	14	→	1 + 4	→	5	
			Divisor		→		1 + 4	→	5	
			Multiply the two values				5 x 5	→	25	
			Add the remainder						12	
			Total						37	
			Cast out nines		→		3 + 7	→	**1**	
Cast out for dividend			1096	→	1 + 0 + 9 + 6	→	165	→	1 + 6	→ **7**

The two values do not match therefore the answer is wrong
— should be 78 r 4

Page 33. DICE DROPPING

If the values on the top of the die are denoted by '*a*', '*b*' and '*c*', then the number produced by entering the three numbers shown on the top of the die and the three on the bottom would be represented by:
$100\,000a + 10\,000b + 1000c + 100(7 - a) + 10(7 - b) + (7 - c)$ which when simplified gives:
$99\,900a + 9990b + 999c + 777$
Removing a common factor of 111 gives
$111(900a + 90b + 9c + 7)$
Dividing by 37 and then by 3 is the same as dividing by 111 which leaves:
$900a + 90b + 9c + 7$
Subtracting 7 leaves
$900a + 90b + 9c$
and dividing by 9 gives
$100a + 10b + c$, the original 3 numbers.

Page 34. DIVISION DECISION I

This Mathematical Novelty is based on the rule for divisibility by 3, which basically states that a number is divisible by 3 if the sum of its digits is divisible by 3. Repeating a two digit number twice means that you will end up with three lots of the original two digit number or three times the digital sum, therefore the 6 digit number formed must be divisible by 3.

Page 36 IT ALL ADDS UP

Adding 9999 (the number formed by combining your number with the student's number) twice, produces 19 998 or 20 000 less 2. The answer is therefore found by placing 2 in front of the original 4 digit number and then taking 2 away from the units digit. This Mathematical Novelty may easily be altered to work with 3 digits, 5 digits etc.

Page 37. THINK OF A NUMBER I

Think of a number	n
Treble it	$3n$
Add ten	$3n + 10$
Double it	$6n + 20$
Subtract 14	$6n + 6$
Divide by 6	$n + 1$
Subtract 1	n

You may create your own "Think of a Number" puzzle using a little Algebra.

Page 37. THINK OF A NUMBER II

Think of a number	n
Double it	$2n$
Add 100	$2n + 100$
Halve it	$n + 50$
Subtract 29	$n + 21$
Take away the number first thought of	21

The finishing number may easily be altered by changing the amount you subtract in the fifth instruction. You may subtract any number from 1 to 49. To work out your first number take the number you use away from 50.
You may also wish to alter the amount added in step 3. Any even number may easily be substituted for 100. To work out your finishing number halve the number you add and subtract the amount used in step 5.

You may wish to create your own "think of a number puzzle".
A few guidelines for developing these types of puzzles may be helpful.
Generally they come in two types.

1. Where you create steps that lead to a specific number at the finish (e.g. Think of a Number II).
2 Where you finish with the number you started with, or a derivation of the number from which you can determine the original number (e.g. Think of Number I).

Write an instruction, test it on some numbers and then attempt to write the instruction algebraically.

e.g.	INSTRUCTION	NUMBER	ALGEBRA
	Think of a Number	13	x

If your algebra is a little rusty, the following reminders may be helpful.

	INSTRUCTION	NUMBER	ALGEBRA
	Double the number	26	$2x$
or	Treble the Number	39	$3x$
or	Add 6	19	$x + 6$
or	Subtract 2	11	$x - 2$

If you add or subtract a number and then multiply, remember to multiply everything.
e.g. If you were to double $x - 2$ you would then have $2x - 4$.

Page 38. SHOWING YOUR AGE

We can track the progress of this "think of a number" type puzzle using a little algebra.
If the month is represented by 'm' and the last 2 digits of the year of birth by 'd', then writing down the month followed by the date would give

$$100m + d$$

Multiplying by 2, 5 and 10 is the same as multiplying by 100 therefore we get

$$10\,000m + 100d$$

Adding the original number

$$\begin{aligned} &10\,000m + 100d \\ &+ 100m + d \\ &= 10\,100m + 101d \end{aligned}$$

The hours, days, weeks etc. total 500 making

$$10\,100m + 101d + 500$$

The digits in the units, tens of thousands and hundred thousand positions remain unaffected. These represent the year and month respectively.

Page 39. DIABOLIC DOMINOES

This Mathematical Novelty is simply a variation of Think of a Number. Instead of the student supplying the number, the domino or die acts as a stimulus.
Let n = Starting Number.
Multiplying by 5 gives $5n$.
Adding 6 produces $5n + 6$.
Doubling the result gives $10n + 12$.
Adding the other number on the domino gives $10n + x + 12$
Subtracting 12 leaves $10n + x$.
As dominoes only contain single digit numbers the first number will be in the tens place. (The instructions, multiply by 5 and then multiply by two ensures this.) The second number will always be in the units place.

Page 40. PICK A CARD– ANY CARD

This is basically the same as multiplying by 10, which in effect alters the place value of the face value of the card so that it ends up in the 10's position.

	e.g.	*General Case*
Pick a card	9 of clubs	a
Double the face value	18	2a
Add 4	22	2a + 4
Multiply by 5	110	10a + 20
Add	2 for a Club. 4 for a Diamond. 6 for a Heart. 8 for a Spade.	

Subtracting 20 leaves the face value in the tens place and the card value in the units place.

The trick may be varied by altering the values added for the suit. These values, however should always be single digits to ensure that the suit value ends up in the units place. e.g 1 for a Club, 2 for a Diamond, 3 for a Heart, 4 for a Spade.

Page 41. MAGIC MONTHS

If the 3 x 3 block of dates is represented by:

d	$d + 1$	$d + 2$
$d + 7$	$d + 8$	$d + 9$
$d + 14$	$d + 15$	$d + 16$

Adding the nine dates

$$d + (d + 1) + (d + 2) + (d + 7) + (d + 8) + (d + 9) + (d + 14) + (d + 15) + (d + 16)$$

gives $9d + 72$ or $9(d + 8)$

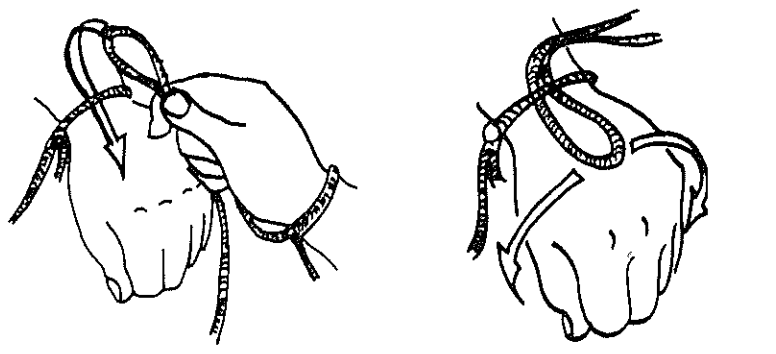
Dividing by nine will give $d + 8$ or the middle date. To work out the surrounding dates simply add or subtract the appropriate amounts.

Page 42. DIVISION DECISION II

This Mathematical Novelty relies on an understanding of the test for divisibility by 9 which states "a number is divisible by 9 if the sum of its digits is divisible by 9".

The variation which creates a number which is divisible by 3 is an application of the rule for divisibility by three which states "a number is divisible by 3 if the sum of its digits is divisible by 3.

Page 43. TIED UP IN KNOTS



Page 44. NUMBER STRIPS

This Mathematical Novelty works because each strip adds up to 20, plus the value shown in the second row.

i.e.

7	3	2	9
6	4	1	8
3	8	7	3
9	4	6	6
1	5	5	2

7 + 3 + 9 + 1 = 20   3 + 8 + 4 + 5 = 20   2 + 7 + 6 + 5 = 20   9 + 3 + 6 + 2 = 20

8	5	3	6
3	0	7	2
4	1	6	8
2	5	9	3
6	9	2	3

8 + 4 + 2 + 6 = 20   5 + 1 + 5 + 9 = 20   3 + 6 + 9 + 2 = 20   6 + 8 + 3 + 3 = 20

Regardless of which strips are chosen they will always add to 20, therefore adding 22 220 (20 000 + 2000 + 200 + 20) to the numbers in the second row will give the answer.

You can make your own set of number strips by listing combinations of numbers that add to 20 on each of eight strips (i.e 4 front and back, leaving the second row free on each strip. Any number may be written in the second position.

5	1	4	6	7	3	2	9
?	?	?	?	?	?	?	?
4	9	6	3	6	5	9	3
3	4	8	3	5	8	7	2
8	6	2	8	2	4	2	6

Several variations are possible. The position of the key digits may be altered from the second row to any other row on the strip.

For younger children it would be appropriate to use 6 strips (i.e 3 front and back) so that 3 digit numbers are formed. In this case in place of 22 220 you would add 2220.

e.g. 3 strips of 4 numbers each

9	3	7
6	8	9
2	7	6
5	9	4

In this case the key row is the third row. The rest of the numbers in the strip add to 20.

Why not make up 3 more strips of your own?

Likewise 10 strips could be used to create 5 digit numbers

Larger strips containing 6 numbers on each strip could be used

e.g.

5	9	7	3
6	3	1	5
7	2	9	9
3	6	0	2
9	7	5	6
3	9	8	7

In this case each strip adds up to 30, plus the value of the number shown in the fourth or key row. If these strips were used 33 330 (i.e. 30 000 + 3000 + 300 + 30) would have to be added to the numbers contained in the fourth row.

Why not make up four more strips of your own?

Page No. 45. MIND READER

This Mathematical Novelty is similar to Finding the Lost Digit contained in the book "Number Novelties"

Following the instructions gives:

$1000a + 100b + 10c + d - (1000d + 100c + 10b + a) = 999a + 90b - 90c - 999d$

or $9(111a + 10b - 10c - 111d)$

Subtracting the smaller number from the larger number produces a multiple of 9. Multiplying this value by any number from 2 to 9999 still produces a multiple of 9. When the digits of any multiple of 9 are added together they also produce a multiple of 9. Therefore to find the missing digit all you have to do is subtract the total of the given digits from the next multiple of 9.

Page 46. MYSTERY MATCHBOX

If the starting number is represented by $10a + b$ then subtracting the sum of the digits, $a + b$ leaves $9a$ which must produce a multiple of 9.

Page 47. LIGHTNING ADDITION

You may recognise this sequence as a Fibonacci Sequence.

If we start with the sequence

a		
b		
$a + b$		
$a + 2b$	i.e. $(a + b) + b$	
$2a + 3b$	i.e. $(a + 2b) + (a + b)$	
$3a + 5b$	i.e. $(2a + 3b) + (a + 2b)$	
$5a + 8b$	i.e. $(3a + 5b) + (2a + 3b)$	→ Seventh Number
$8a + 13b$	i.e. $(5a + 8b) + (3a + 5b)$	
$13a + 21b$	i.e. $(8a + 13b) + (5a + 8b)$	
$21a + 34b$	i.e. $(13a + 21b) + (8a + 13b)$	
$55a + 88b$		

11 (Seventh Number) = Total.

$11(5a + 8b) = 55a + 88b$

To multiply a 2 digit number by 11 simply take the two digits of the original number to form the first and last digits of the answer. The middle digit(s) is found by adding the two digits.

eg **36 x 11**

STEP 1 **3 _ 6** **3 + 6 = 9**

STEP 2 **396**

When the two digits add up to more than 9 carry a one to the next number.

eg **89 x 11**

8 _ 9

979

8 + 9 = 17
So carry the one and add it to the eight

A Fibonacci sequence of numbers is one in which the third number is the sum of the first two, the fourth is the sum of the second and third number and so on. Put simply, a term in the sequence is found by adding the two previous terms. While the sequence shown in "Lightning Addition" is a Fibonacci sequence it is not ***the*** Fibonacci sequence which begins with 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89...

Page 48. SUPER SEQUENCES

The rule for working out the sum of the numbers is a sequence starting at one and increasing by one each time is

$$\frac{n(n+1)}{2}$$

i.e. Multiply the last number in the sequence by one more than the number and then halve the result.

After trying this Mathematical Novelty a few times you may wish to inject some history into the lesson by discussing the famous Mathematician called Gauss. As the story goes Gauss was a bright but very troublesome student so one day his teacher decided to keep young Gauss quiet for a while. He set him the task of adding all the whole numbers from 1 to 100. There were no calculators back in that time, so the teacher expected Gauss to add the numbers manually. Imagine the surprise on the teacher's face when in less than a minute Gauss returned with the answer. Here is how he did it. He noticed the following pattern.

$$1 + 2 + 3 + \dots + 98 + 99 + 100$$

Each pair of numbers added to 101. There were 50 pairs, therefore the total was 5050.

Variation

To find the sum of consecutive odd numbers in a sequence

- Add one to the largest number.
- Halve it.
- Square it.

To quickly find the sum of consecutive even numbers.

- Halve the largest number.
- Multiply the result by one more than itself.

Page 49. ALL MATHS IS WONDERFUL

This Mathematical Novelty only works when the top cut from the deck of cards contains between 20 and 29 cards. The difference between the number of cards in this part of the deck and the sum of the digits of this "top set number" will always be 18. The phrase "ALL MATHS IS WONDERFUL contains 19 letters and therefore will always bring you back to the magic card.
A little algebra explains why this works.
Let '*a*' represent the tens digit.
Let '*b*" represent the units digit
The two digit number would be $10a + b$
The sum of the digits would be $a + b$
The difference is $9\,a$.
When the number is in the 20's "a" = 2, and the difference is therefore 18. The 19 letter phrase reveals the actual card

Variations

Alter the phrase to be spelled out. Any 18 or 19 letter phrase will do.If an 18 letter phrase such as "YOU LOVE MATHEMATTICS" is used simply count 18 cards and turn the nineteenth card to reveal the magic card.
If the number of cards contained in the cut from the top of the deck contains between 30 and 39 cards then the difference between number of cards in the top cut from the deck and the sum of the digits of this "top set number" will always be 27 (i.e. 9 x 3). Likewise if the number of cards contained in the top cut from the deck is between 10 and 19 then you will only need to count 9 cards (i.e. 9 x 1) to reveal the magic number.

ISBN 0 646 19424 0