NUMBER

NOVELTIES

A Set of practical activities designed to motivate the teaching and learning of number.

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Practical activities designed to motivate the teaching and learning of number.

Preface

All of us at one time or another have heard the cry of anguish or seen the despondent look on a child's face when we announce it is time for maths. Let's face it, Mathematics is probably the most hated subject in school – and yet it doesn't have to be.

This book is designed for teachers who are looking for ways to motivate their students and for all those children who hate mathematics. Inside you will find tricks and puzzles designed to be used by teachers with their students or by students on their own. Teachers will also find some Mathematical Magic designed to liven up any maths lesson.

The activities are always fun, provide a challenge and often call for considerable calculation, thereby providing children with painless practice. More importantly, the solutions often depend on properties of numbers, the investigation of which may lead to important mathematical understandings.

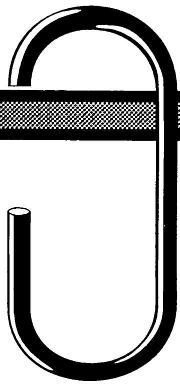
More able students will often appreciate *why* a trick works once they understand *how* it works. Many of the Number Novelties may be solved by the application of simple algebra, providing an informal introduction to algebra for lower secondary students.

Students will be amazed by the Mathematical Magic tricks contained in the book. Teachers will be grateful for the answers and detailed explanations given.

Number Novelties will help children see the magic of mathematics and help them appreciate its power and beauty.

Contents

FUN WITH FIGURES



A NOTE TO TEACHERS

The activities in this section may be used by the teacher to motivate children, or they may be copied onto card to enable children to pursue topics which interest them.
The symbols on the top right hand page indicate the arithmetic function covered.

+ − X

1089

Choose a 3 digit number where the hundreds digit is at least two more than the units digit. **e.g. 461**

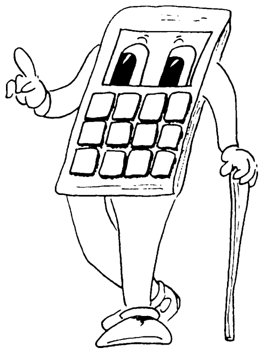
Reverse the digits and then subtract. **− 164**
= 297

Reverse the digits of the answer and then add this number to the answer.
792
+ 297
= 1089

Try another 3 digit number and see what happens.

The number 1089 has some interesting properties.
Try these multiplications.

1089 x 1 =
1089 x 2 =
1089 x 3 =
1089 x 4 =
1089 x 5 =
1089 x 6 =
1089 x 7 =
1089 x 8 =
1089 x 9 =



Write about any patterns you notice.

+ − X

1089 REVISITED

- Write down any five digits in descending order. **e.g. 97432**
- Reverse the digits. **23479**
- Subtract the smaller number from the larger.

97432
− 23479
73953

- Reverse the digits of the answer and then add this new number to the answer.

73953
+ 35937
109890

- Try another 5 digit number and see what happens.

Do you think you will always arrive at the same answer?

Try some more.

Try these multiplications and write about any patterns you notice.

9 x	1089	=	4 x	2178	=
9 x	10989	=	4 x	21 978	=
9 x	109 989	=	4 x	219 978	=
9 x	1 099 989	=	4 x	2 199 978	=
9 x	10 999 989	=	4 x	21 999 978	=

TAUTONYMS

A *tautonym* is a number where the digits are repeated twice in sequence. For example 55, 7272 and 397397 are all tautonyms.

• Choose any 3 digit number	**e.g. 295**
• Repeat it *i.e. enter its tautonym*	**295295**
• Divide by **7**	**42185**
• Divide by **11**	**3835**
• Divide by **13**	**????**

Compare your answer with your original number. What do you notice?

Try again using a different 3 digit number.

Will this always happen? Why?

Does it matter whether you change the order and divide by 13, 11, and then 7? Try it to find out if you are not sure.

÷

TRIPLE TREAT

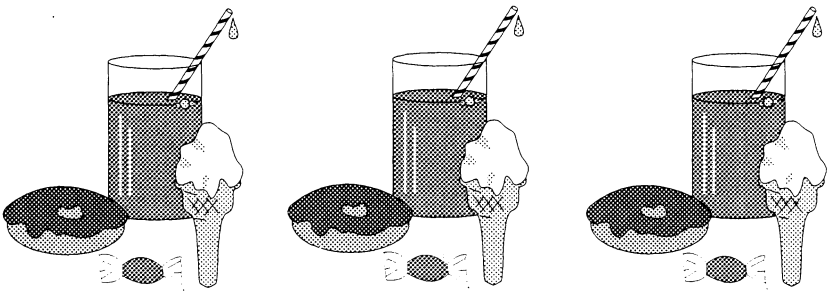
• Choose a 2 digit number	e.g.	39
• Repeat twice		393939
• Divide by **13**		30303
• Divide by **21**		1443
• Divide by **37**		????

Compare your answer with your original number.
What do you notice?

Try again using a different 2 digit number.

Will this always happen? Why?

Does it matter whether you change the order and divide by 37, 21 and then 13? Try it to find out if you are not sure.



4 DIGIT MADNESS

- Write down any 4 digit number where all four digits are different. **e.g.** **5643**
- Re-arrange the 4 digits to produce:
  (a) the largest number you can **6543**
  (b) the smallest number you can. **3456**
- Subtract the smaller number from the larger number.
- Now you should have a new number.

  6543
  – 3456
  = 3087
- Do the same to this number.

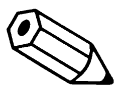
  8730
  – 0378
  = 8352
- Continue doing this until you reach 6174.

  8532
  – 2358
  = 6174

Try this using your own 4 digit numbers. It shouldn't take more than 7 tries to reach 6174.

Try to find starting numbers that produce 6174 in one, two, three and four tries.

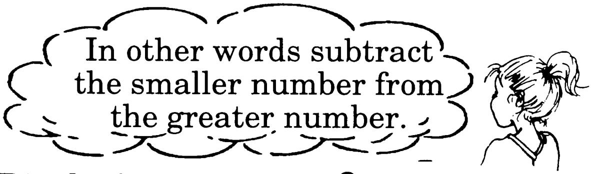
What happens when you start with 6174?
Will the trick still work if you start with a number where all four digits are not different?



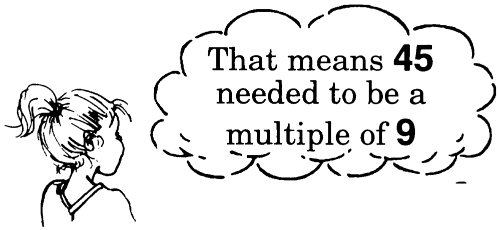
Try 1355, 5544, 2333, 3333

MULTIPLES OF 9

- Write down any 2 digit number. **e.g. 61**
- Reverse the digits. **16**
- Find the difference. **61 − 16 = 45**



- Divide this number by **9**. **45 ÷ 9 = 5**



Try starting with other 2 digit numbers.

1. *Do you always get multiples of 9?*
2. *Can you get all of the multiples of 9 from 9 to 81?*
3. *Can you find any connection between the number you start with and the multiple of 9 you end up with?*

Try to explain the relationship.

CRISS CROSS I

- Choose any number between **50** and **99.** **e.g. 85**
- Add **62** to this number. **62 + 85 = 147**
- Cross off the left hand digit of the result and add it on to the units digit. **~~1~~47 ➡ 48**
- Subtract the result from your original number.

$$\begin{array}{r} 85 \\ -\ 48 \\ \hline =\ 37 \\ \hline \end{array}$$

Try it again.

What do you notice?

CRISS CROSS II

- Choose any *two* numbers between **50** and **100**. — **e.g. 59 and 86**

- Add them.

  59
  + 86
  = 145

- Cross out the left hand digit. — **~~1~~45**

- Add one to the remaining number. — **45 + 1 = 46**

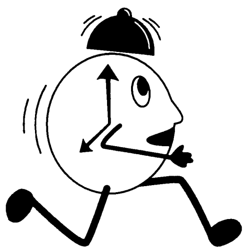
- Subtract the result from your original number.

  145
  – 46
  = 99

Try again using two different numbers.

What do you notice about the answer?

Does it work if you use 50 and 100 as your starting numbers?



THE MISSING EIGHT

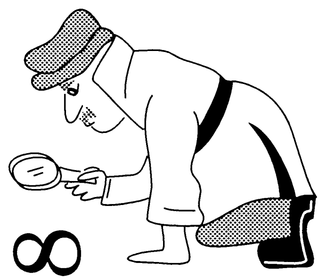
- Write down the numbers **1** to **9** but leave out the **8**. — **e.g. 12345679**
- Choose one of the eight numbers. — **4**
- Multiply this number by **9.** — **4 x 9 = 36**
- Multiply your answer by **12 345 679.** — **???**
- *What do you notice about your answer?*

Try the same procedure but this time choose a different number to multiply by **9**.

Can you predict what the answer will be for any multiple of 9?

Try and get every number from 1 to 9 excluding 8.

Why has the 8 been left out?



+ − x ÷

SQUARE

Think of a number.						8
• Add the number to itself.	8	+	8	=	16
• Subtract it from itself.	8	−	8	=	0
• Multiply it by itself.	8	x	8	=	64
• Divide it by itself.	8	÷	8	=	1
• Add your four answers together.				=	81

What do you notice about **81**?

$81 = 9^2$ or $(8 + 1)^2$

Try the same procedure with seven.

• Add the number to itself.	7	+	7	=	14
• Subtract it from itself.	7	−	7	=	0
• Multiply it by itself.	7	x	7	=	49
• Divide it by itself.	7	÷	7	=	1
• Add your four answers together.				=	64

Note also that: **$(7 + 1)^2$ or $8^2 = 64$.**

Try some more numbers to see if this is always the case.

A TRICK FOR SQUARES

• Choose a number.	**23**
• Square it (multiply it by itself).	**23 x 23 = 529**
• Add one to your original number.	**23 + 1 = 24**
• Square it.	**576**
• Subtract the smaller number from the larger number.	**576 – 529 = 47**
• Subtract one.	**47 – 1 = 46**
• Halve the number.	

What do you notice?

Try this for several starting numbers. You may need a calculator to help you.

What always happens?

PALINDROMES

A **PALINDROME** is a word or number that can be read the same both forwards and backwards.
e.g. **MUM, DAD, NOON, RADAR, 99, 323, 747, etc.**

There are even some palindromic sentences.

MADAM I'M ADAM.
A MAN, A PLAN, A CANAL, PANAMA.
ABLE WAS I ERE I SAW ELBA

You can produce your own number palindromes using the following procedure:

Select a number	**18**
Reverse the digits	**81**
Add the numbers	**99**

In one step (reversing and adding) we have formed a palindromic number. The number **28** requires two steps.

$$\begin{array}{r} 28 \\ +\,82 \\ 110 \end{array} \quad \text{and} \quad \begin{array}{r} 110 \\ +\,011 \\ 121 \end{array}$$

You will find most numbers form a palindromic number in very few steps.

Try these **123, 632, 458, 184, 291, 79.**

Try some of your own.

POINTED PALINDROMES

A palindrome is a word or number that can be read the same both forwards and backwards.
Decimal palindromes may be produced using the following procedure.

Select a decimal	**5·13**
Reverse the digits	**31·5**
Add the numbers	**36·63**

It is important that the decimal point be placed in the correct position otherwise a palindrome is not formed. Some numbers require more than one step:

	21·9
Reverse and add	**+ 9·12**
	= 31·02
	31·02
Reverse and add	**+ 20·13**
A palindrome is formed	**= 51·15**

You will find most numbers form a palindromic number in very few steps.

Try these: **15·4, 25·4, 16·7, 64·7, 98·3**

What happens when you start with a palindrome?

e.g. **21·12, 16·61, 17·71, 51·15, 36·63**

DIGIT SHUFFLE

- Choose a two digit number, where each digit is different. **e.g. 27**
- Write down all the numbers that may be formed by altering the positions of the digits. **27, 72**
- Add them up **99**
- Find the sum of the digits in the *original* number. **2 + 7 = 9**
- Divide the total by the sum of the original digits **99 ÷ 9 = 11**

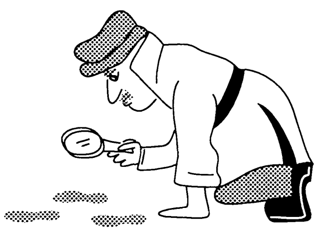
Try it again using a different two digit number.

What do you notice?

Try some three digit numbers, where each digit is different.

Do you always get the same result?

What do you think the result will be for four digit numbers? Test it!



What happens if some of the digits are the same?

SCRAMBLED DIGITS

- Choose any five digit number. **e.g. 56 134**

- Scramble the digits. **34 165**

- Subtract the smaller number from the larger.

$$\begin{array}{r} 56\,134 \\ -\ 34\,165 \\ \hline =\ 21\,969 \\ \hline \end{array}$$

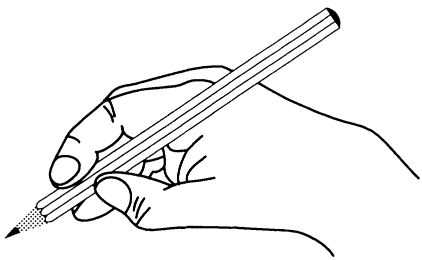
- Add the digits that form the answer until you reach a single digit.

$$2 + 1 + 9 + 6 + 9 = 27$$
$$2 + 7 = 9$$

Try it again using your own five digit number.

What do you notice?

Try starting with 2, 3, 4 and 6 digit numbers and write down what happens.



DOUBLE DIGIT DILEMMA

•	Choose a two digit number where no digit is repeated.	**e.g. 27**
•	Reverse the digits.	**72**
•	Subtract the smaller from the larger.	**72** **– 27** **= 45**
•	Find the difference between your original two digits.	**7 – 2 = 5**
•	Divide your answer by this difference.	**45 ÷ 5 = 9**

Try again using a different two digit number.

What do you notice?

FINDING THE LOST DIGIT

- Write down any three digit number. **e.g 623**

- Cross out one digit, but ***not*** a **0** to leave a two digit number. **6~~2~~3** → **63**

- Subtract the two digit number from the three digit number. **623 – 63 = 560**

- Add up the digits of the remaining number until a single digit number remains. **5 + 6 + 0 = 11**, **1+ 1 = 2**

- Try it again using a three digit number of your own choosing.

What do you notice?

WHO KNOWS?

- Choose a three digit number. Do not choose one which uses the same digit three times **e.g. 437**
- Use the same three digits to form the largest 3 digit number that you can. **743**
- Now form the smallest 3 digit number that you can. **347**
- Find the difference between the largest and the smallest number.

$$\begin{array}{r} 743 \\ -\ 347 \\ \hline 396 \end{array}$$

- Repeat the process using the three digits from your new number.

$$\begin{array}{r} 963 \\ -\ 369 \\ \hline 594 \end{array}$$

- Repeat the process one more time using the three digits from your new number.

$$\begin{array}{r} 954 \\ -\ 459 \\ \hline 495 \end{array}$$

Try using **747.** *What do you notice?*

Try the same procedure with other 3 digit numbers.

What do you notice?

TRIPLE DIGIT DIVISION

- Choose any three digit number where all three digits are the same. **e.g. 555**
- Add the three digits. **5 + 5+ 5 = 15**
- Divide your original number by your answer. **555 ÷15**

What is your answer?

Try again using a different 3 digit number.

What do you notice?

Do you think this will occur with all three digit numbers where all three digits are the same?

Try it.

111,	**222,**	**333,**
444,	**555,**	**666,**
777,	**888,**	**999**

SEEING DOUBLE

- Write down any three single digit numbers.

e.g. 4, 6, 1

- Use these three numbers to make six 2 digit numbers.

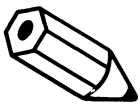
46, 41, 61, 64, 14, 16

- Add the six two digit numbers. **242**
- Add the original three numbers. **4 + 6 + 1 = 11**
- Divide the larger number by the smaller.
- Write down your answer.
- Try this again using three different single digit numbers.
- *What do you notice?*

Try using consecutive single digit numbers.

1, 2, 3　　2, 3, 4　　3, 4, 5　　4, 5, 6
5, 6, 7　　6, 7, 8　　7, 8, 9

- *What do you notice?*



OTHER METHODS OF CALCULATION

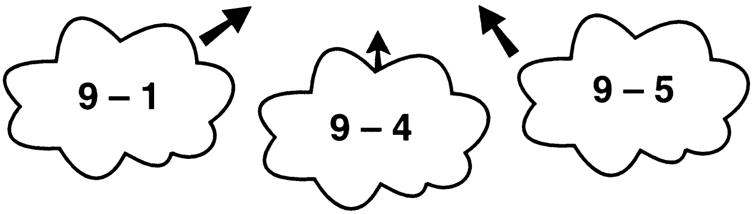
SURPRISING SUBTRACTIONS I

To perform the following subtraction

$$\begin{array}{r} 431 \\ -\,145 \end{array}$$

- Subtract each digit of the second line from 9 to form

i.e. **8 5 4**

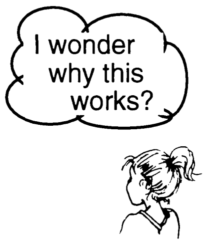


- Add the result to the first part of the subtraction

$$\begin{array}{r} 431 \\ +\,854 \\ \hline 1285 \\ \hline \end{array}$$

- Cross off the 1 from the left of the answer and add it to the units.

1285 ➡ 286



- Check to see whether this is correct.

- *Try these*

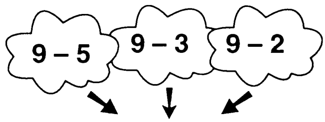
741	**–**	**532**	**617**	**–**	**468**
635	**–**	**246**	**1647**	**–**	**864**

SURPRISING SUBTRACTIONS II

To perform the following subtraction

$$\begin{array}{r} 532 \\ -\ 267 \end{array}$$

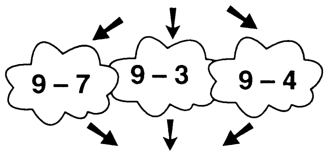
- Subtract each digit of the first line from **9** and then add



$$\begin{array}{r} 4\ 6\ 7 \\ +\ \underline{2\ 6\ 7} \\ \underline{\underline{7\ 3\ 4}} \end{array}$$

- Now take each digit of the answer away from 9.

7 3 4



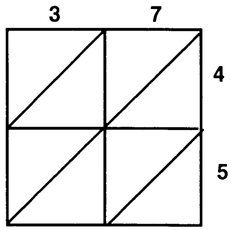
2 6 5

- Check to see whether this answer is correct.
- *Try these* **467 – 348,** **643 – 258**

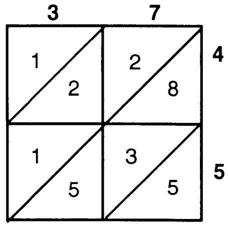
  767 – 555, **841 – 394**
- Check using your normal method or by calculator.

VENETIAN GRID METHOD

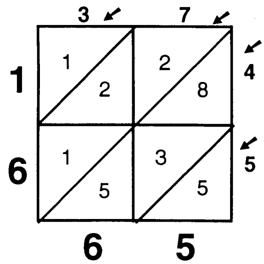
To multiply using this method you need to draw a grid. For example, to multiply 37 x 45 draw the diagram on the right.



Complete each square by multiplying the two numbers above and to the right of the square. In each square write the tens above the diagonal, and the units below.



Starting from the bottom right corner, add numbers between the diagonal and carry to the next column where necessary. The answer is read from the top left to the bottom right.



Try using this method to calculate:

57 x 26, 43 x 24, 71 x 156 273 x 368

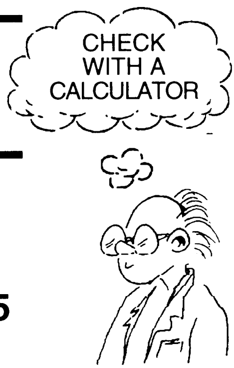
Check your answers using another method.

RUSSIAN PEASANT METHOD

All you need to know to use this method is how to double and halve numbers.
Double one number, halve the other, leaving out halves, until you reach **1** in the halving column.
Examine the halving column in the example below.
Whenever an even number appears, cross out the number opposite in the doubling column.
Add the remaining numbers in the doubling column. This will give the desired product.

e.g **37 x 45**

D	H
37	45
~~74~~	22 ←
148	11
296	5
~~592~~	2 ←
1184	1
1665	

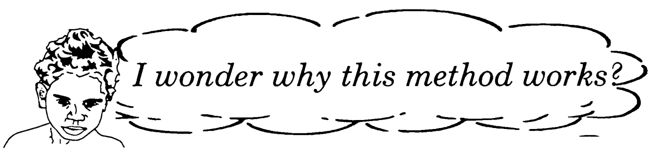


i.e. 37 x 45 = 1665

Try using this method to calculate:

34 x 28, 43 x 24, 57 x 36,
69 x 73, 84 x 31, 79 x 64.

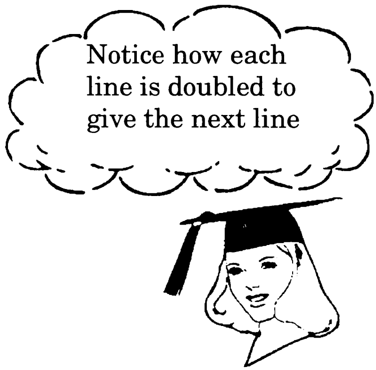
Check your answers by another method.



MUMMY MULTIPLICATION.

The Egyptians (c 1600 BC) multiplied numbers such as **23** x **18** using the following procedure:

	A				B
	1	x	18	=	18
so	2	x	18	=	36
so	4	x	18	=	72
so	~~8~~	~~x~~	~~18~~	=	~~144~~
	16	x	18	=	288
					414



In the above example we stopped at **16** because this is the *largest number below* the figure by which they were multiplying. They would then look for a combination that added to **23**. (In the above example **16 + 4 + 2 + 1** equals **23.**) The remaining numbers are crossed out in all columns. The numbers left in column B are then added to find the answer.

Try using this method to multiply:

28 x 17, 24 x 22, 35 x 41, 48 x 32, 59 x 43

Check your answers using another method.

JUST A LITTLE LESS THAN 100

This shortcut may be used to multiply two numbers that are near 100.

To multiply 96 by 87 try using the following procedure:

96 (100 – 96 = 4)
x 87 (100 – 87 = 13)

Find the difference between each number and 100

The first two digits of the answer are found by subtracting 13 from 96

96 – 13 = 83

83 _ _

The last two digits are found by multiplying 4 by 13

Multiply the differences to determine the last two digits of the number, e.g. 4 x 13 = 52.

83**52**

Try these using the shortcut shown above

93 x 86,	89 x 91,	98 x 56,	97 x 79
94 x 84,	89 x 97,	96 x 82,	94 x 87

Check your answers using another method.

A PAPER CALCULATOR

NAPIER'S RODS

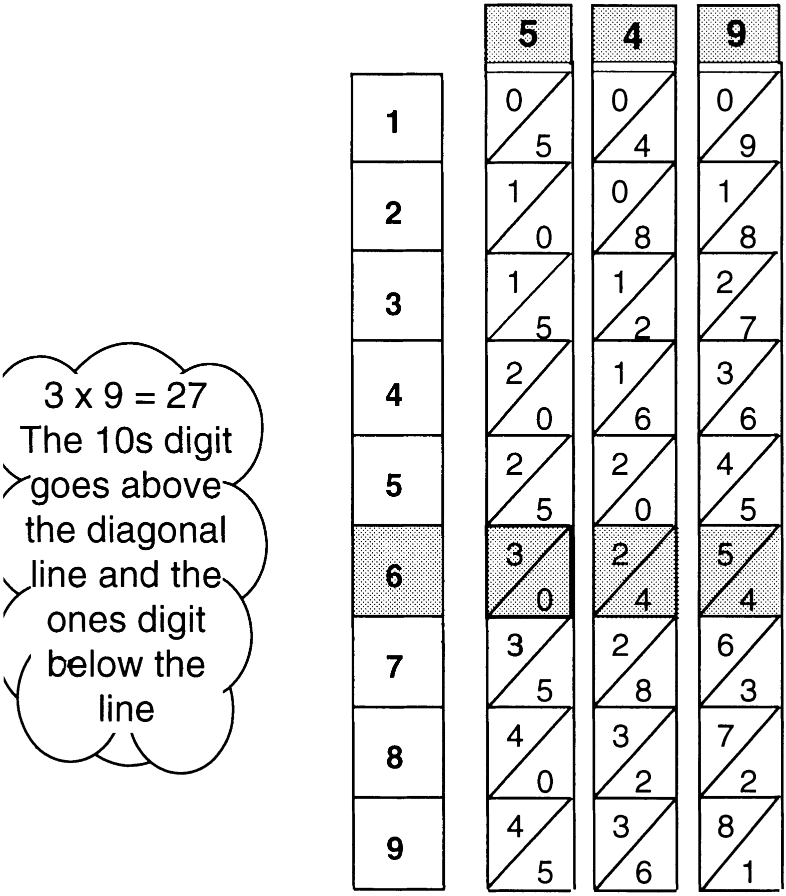
John Napier was born in 1550 and died in 1617 long before the invention of calculators. He developed his own form of calculator which is now known as Napier's Rods.
To make your own "paper calculator" you need to construct a set of 10 strips containing all the tables.
The 9 x strip has been done for you. Complete the rest.

x	0	1	2	3	4	5	6	7	8	9
1	0/0	0/1	0/2	0/3						0/9
2	0/0	0/2	0/4	0/6						1/8
3	0/0	0/3	0/6							2/7
4	0/0	0/4								3/6
5	0/0	0/5								4/5
6	0/0	0/6								5/4
7	0/0	0/7								6/3
8	0/0	0/8								7/2
9	0/0	0/9								8/1

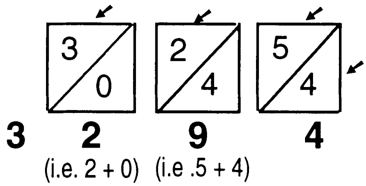
The rods can be used to calculate in the following manner.

e.g. 6 x 549

Pick out the 4, 5 and 9 rods and place them in correct order



The answer to 6 x 549 is given in row 6. Simply add the numbers between the diagonal lines



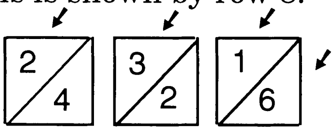
Try using the rods to calculate **7 x 634, 4 x 358, 6 x 947.**

You can use your rods to calculate answers to harder questions.

For example, to perform the following multiplication,

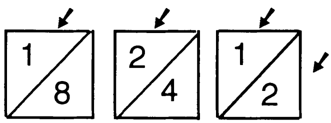
68 x 342

pick out the 3, 4 and 2 rods and place them in order. First multiply 342 by 8. This is shown by row 8.

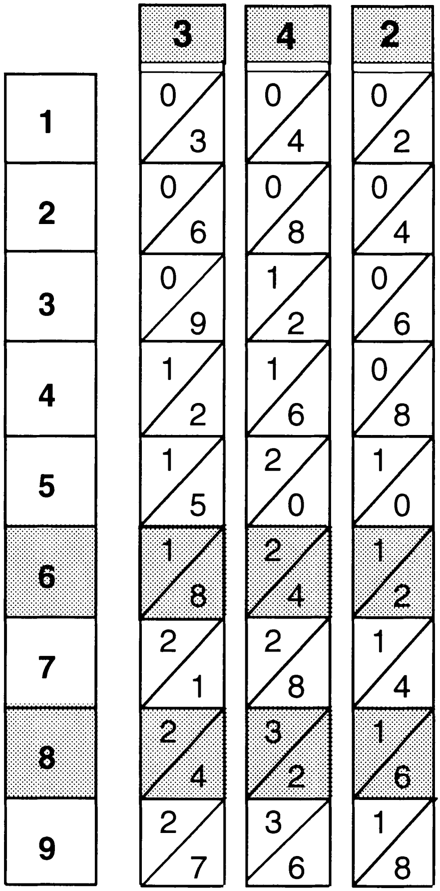


2 7 3 6

Then multiply 342 by the digit in the 10's place (i.e. 6)



2 0 5 2



If **6 x 342 = 2052**, then **60 x 342 = 20520**

Add the two parts to find the answer.

$$\begin{array}{r} 20520 \\ +\ 2736 \\ \hline 23256 \\ \hline \end{array}$$

Try these: **47 x 368,** **35 x 412,** **82 x 674,** **93 x 176**

MULTIPLICATION SHORTCUTS

To square a two digit number ending in five, multiply the tens digit by one more than itself, then place 25 at the end of your answer.

e.g.

35^2

3 x 4   **5 x 5** is always **25**

1225

Try using this method to square all of the two digit numbers that end in 5.

15, 25, 35, 45, 55, 65, 75, 85, 95.

Check your answers using another method.

x 11

MULTIPLICATION SHORTCUTS

To multiply a 2 digit number by 11 simply take the two digits of the original number to form the first and last digits of the answer. The middle digit(s) is found by adding the two digits.

eg **36 x 11**

STEP 1 **3 __ 6**

STEP 2 **396**

3 + 6 = 9

When the two digits add up to more than 9 carry a one to the next number.

eg **89 x 11**

8 __ 9

979

8 + 9 = 17
So carry the one and add it to the eight

Try these: 35 x 11, 44 x 11, 61 x 11,
52 x 11, 48 x 11, 78 x 11.

MULTIPLICATION SHORT-CUTS

In order to use this shortcut you must remember that

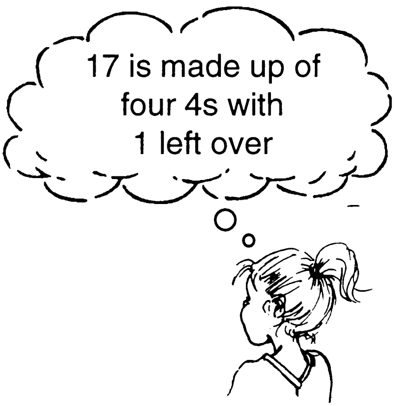
4 x 25 = 100

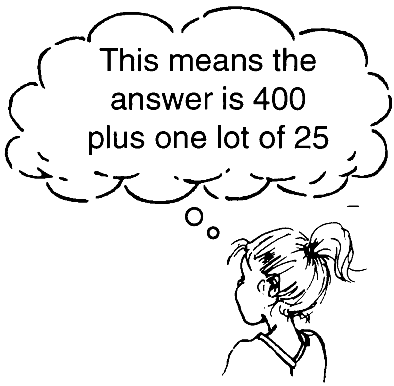
The rest is easy.

For example, to multiply

17 x 25

simply work out how many fours there are in 17. This tells you the number of hundreds in the answer. Then add the remaining number of twenty fives to work out the answer.





Try these **23 x 25, 33 x 25, 38 x 25, 47 x 25**

x 99

MULTIPLICATION SHORT-CUTS

When multiplying a number by **99** simply multiply by 1 0 0 and subtract one lot of the number.

e.g. **24 X 99**

is the same as **(24 X 100) – 24**

$$\begin{array}{r} 2400 \\ -\ 24 \\ \hline 2376 \end{array}$$

Try these **38 x 99** **42 x 99** **81 x 99**

x 101

When multiplying a number by **101** simply multiply by **100** and add one lot of the number

e.g. 24 x 101

is the same as **(24 X 100) + 24**

$$\begin{array}{r} 2400 \\ +\ 24 \\ \hline 2424 \end{array}$$

Try these **29 x 101** **57 x 101** **84 x 101**

CLASSROOM MAGIC

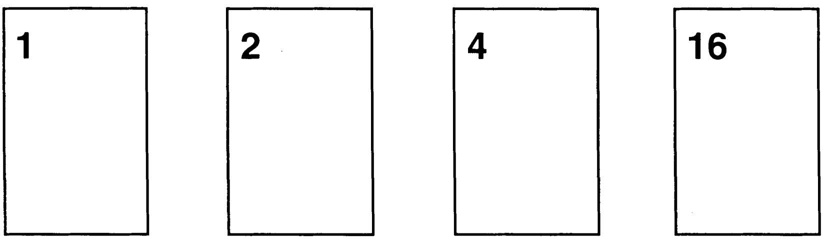
The ideas in this section require input from the teacher. A few props and a theatrical approach will greatly enhance the motivational value of each idea.
The material is this section may not be reproduced.

MIND READING TRICK

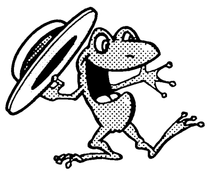
Prepare a set of six cards, similar to those shown on the next page.

- Ask a student to choose a number between 1 and 63. This number should be kept secret.
- Show the student each card in turn and ask them whether their number appears on that particular card.
- If the student says that their number appears on the card note the number in the top left hand corner of the card. Add these numbers as you go.
- The total is the secret number.

For example the SECRET NUMBER: 23, appears on the following cards



1 + 2 + 4 + 16 = 23



1	23	45
3	25	47
5	27	49
7	29	51
9	31	53
11	33	55
13	35	57
15	37	59
17	39	61
19	41	63
21	43	

2	23	46
3	26	47
6	27	50
7	30	51
10	31	54
11	34	55
14	35	58
15	38	59
18	39	62
19	42	63
22	43	

4	23	46
5	28	47
6	29	52
7	30	53
12	31	54
13	36	55
14	37	60
15	38	61
20	39	62
21	44	63
22	45	

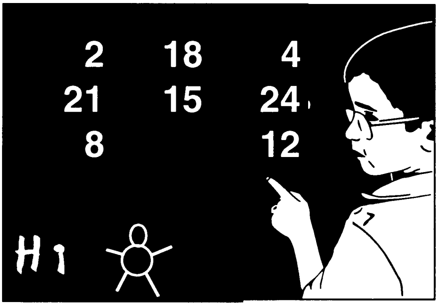
8	27	46
9	28	47
10	29	56
11	30	57
12	31	58
13	40	59
14	41	60
15	42	61
24	43	62
25	44	63
26	45	

16	27	54
17	28	55
18	29	56
19	30	57
20	31	58
21	48	59
22	49	60
23	50	61
24	51	62
25	52	63
26	53	

32	43	54
33	44	55
34	45	56
35	46	57
36	47	58
37	48	59
38	49	60
39	50	61
40	51	62
41	52	63
42	53	

NUMBER SPELLING

Write the following set of numbers on the board or have them written on a poster.
The order is not important.



- Ask one of your students to choose a number while you turn your back.

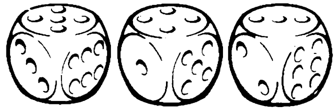
- Ask the rest of the class to spell out the number in their heads while you point to the various numbers on the board.

- They should spell a letter to themselves every time you point to a number.

- The order in which you point to the numbers is important. *See the answers and explanations for the technique to use.*

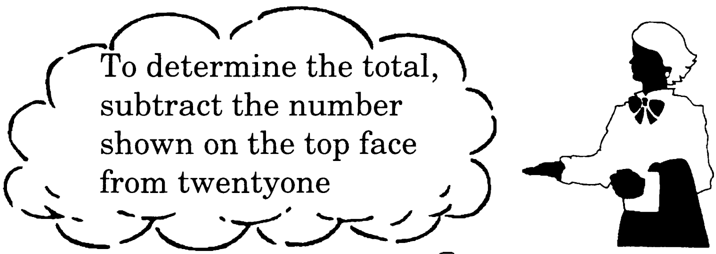
- Instruct the class to call out stop when they finish spelling the word. You should find that you are pointing at the secret number.

THRICE DICE

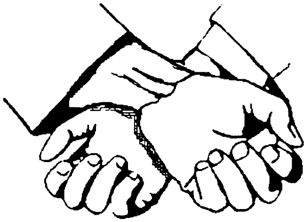


You will need 3 dice
and a pen and paper.

- Ask a volunteer from the audience to roll the dice, while you stand facing away from the audience.
- Choose a second volunteer to pick up the 3 dice in any order and stack them on top of each other. You remain facing away from the audience.
- Instruct the 2 volunteers to add up (silently) the numbers of the *five hidden faces* of the dice as they stack them up.
- When they have added the numbers ask them to write the total down on the paper provided and show it to the audience.
- Ask the volunteers to tear up the paper.
- Look at the stack of dice and note the top face on the stack.
- Announce the total to the class after some show of consternation (and some simple mental addition).

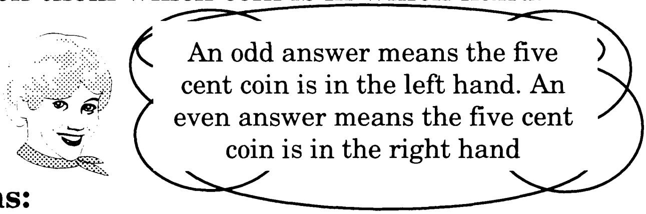


WHICH HAND?



You will need a ten cent coin
and a five cent coin

- Select a volunteer from the audience.
- Hand the two coins to the volunteer and tell them to place one coin in each hand.
- Have them show the audience which coin is in which hand while you turn your back.
- Ask the volunteer to multiply what is in their right hand by 4. (Give them a little time.) Ask them **not** to tell you the answer.
- Tell them to multiply the coin in their left hand by 3. Ask them **not** to tell you the answer.
- Tell them to then add the two numbers together. This time they can tell you the answer.
- Now tell them which coin is in which hand.



Variations:

Instead of multiplying by 4 ask the volunteer to multiply by any even number.
Instead of multiplying by 3 ask the volunteer to multiply by any odd number.

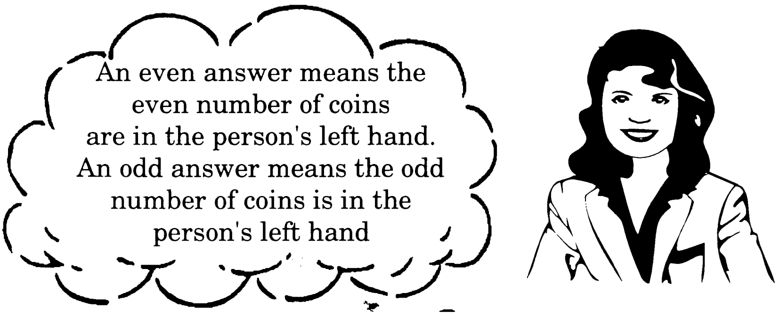
WHICH HAND II?

Ask your student(s) to perform the following steps.

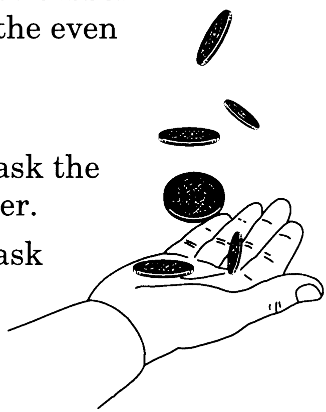
- Divide five coins between your two hands, but do not let me see how many you have put in either hand.

Obviously there will be an odd number of coins in one hand and an even number in the other hand.

- Multiply what you have in your right hand by 2.
- Multiply what you have in your left hand by 3.
- Add the two results and tell me the total.



- State which hand contains the odd number of coins and which hand contains the even number of coins.



Variations:

Instead of multiplying by 2 you could ask the student to multiply by any even number.

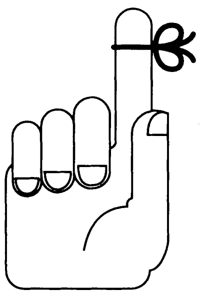
Instead of multiplying by 3 you could ask the student to multiply by any odd number.

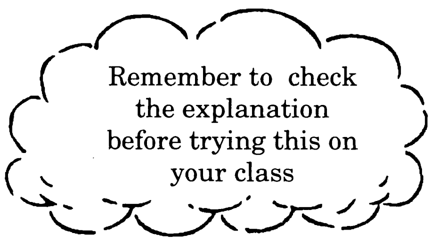
Start with a different number of odd coins e.g. 7, 9 etc.

NUMBER E.S.P.

Ask your student(s) to perform the following steps

• Write down a three digit number	e.g.	611
• Reverse the number		116
• Subtract the smaller number from the larger.		611
		−116
		495





Ask each student in turn to tell you the right hand digit of their answer. Then offer to tell them what their answer is.

Two Digit Variation suitable for younger students.

- Write a two digit number.
- Reverse the digits.
- Find the difference between the two numbers.
- Ask the children to give the last digit of their answer.
- Supply the answer by subtracting given digit from **9** to determine the tens digit.

THE MISSING NUMBER

Ask your student(s) to perform the following steps. 

- Write down any 4 digit number where all the digits are different. **e.g. 6784**

- Add all the digits. **6 + 7 + 8 + 4 = 25**

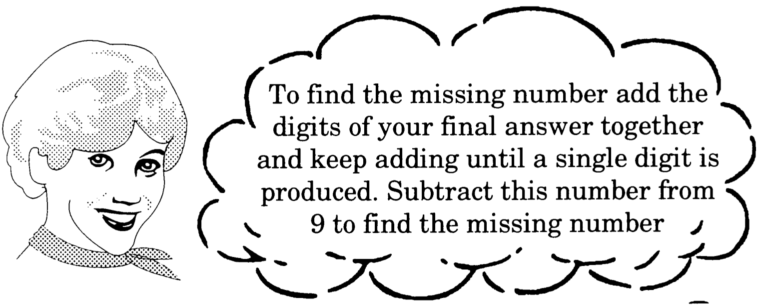
- Keep adding the digits of your sum until you reach a single digit number. This is called the **reduced number.** **2 + 5 = 7**

- Cross out one of the original digits. **6 7 ~~8~~ 4**

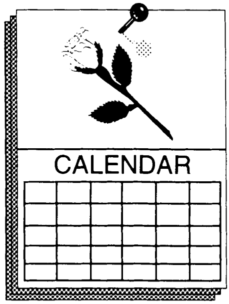
- Subtract the reduced number from the remaining three digit number.

$$\begin{array}{r} \mathbf{674} \\ \mathbf{-7} \\ \hline \mathbf{667} \\ \hline \end{array}$$

Ask the student(s) for their final answer.
Now announce that you will find the number that they crossed out.

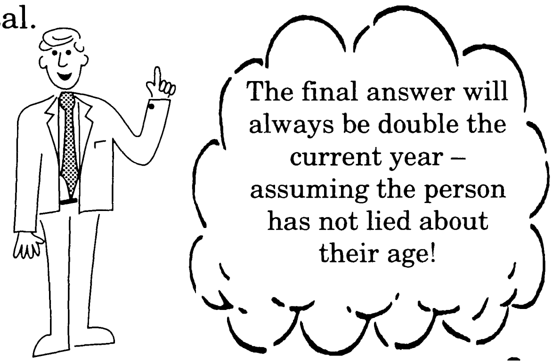


GREAT DATES



Ask your student(s) to perform the following steps.

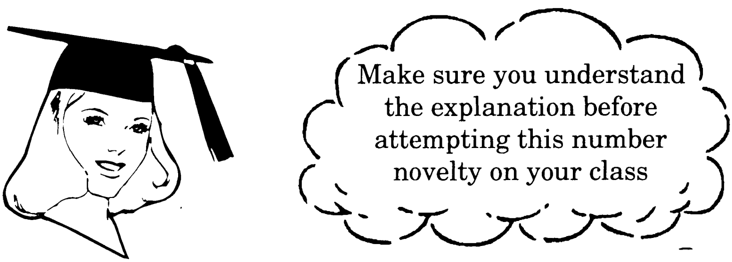
- Write down the year in which you were born. **e.g. 1983**
- Write down the year in which a significant event took place in your life. **1989**
- Write down your age at the end of the current year (Dec. 31). **10**
- Write down the number of years that have passed since the important event occurred. **4**
- Add all the numbers that you have written down.
- Without looking at any of their calculations, announce their final total.



DIGIT DETECTION

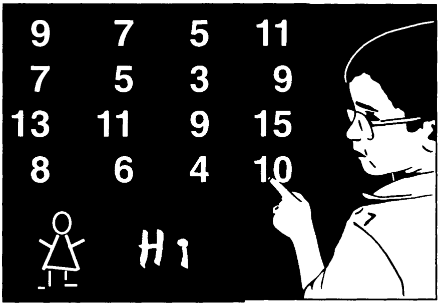
Ask your student(s) to perform the following steps

- Write down any 3 digit number (don't show the teacher). **617**
- Add the digits. **6 + 1 + 7 =14**
- Subtract the sum of the digits from your three digit number. **617 –14 = 603**
- Ask the students to tell you the first two digits of their answers. **6 and 0**
- Offer to tell the students what their final answers were.



TRICKY TABLE

- Draw the following table on the board or have it drawn on a poster.



- Ask the students to make their own copy.
- Give the following instructions:

1. Circle any number and cross out any other numbers in the same column or row as the circled number.

9	7	5	~~11~~
7	5	3	~~9~~
13	11	9	~~15~~
~~8~~	~~6~~	~~4~~	(10)

2. Circle any number that has not been crossed out or circled and then cross out all the remaining numbers in the same row or column.

(9)	~~7~~	~~5~~	~~11~~
~~7~~	5	3	~~9~~
~~13~~	11	9	~~15~~
~~8~~	~~6~~	~~4~~	(10)

3. Repeat step 2.

(9)	~~7~~	~~5~~	~~11~~
~~7~~	(5)	~~3~~	~~9~~
~~13~~	~~11~~	9	~~15~~
~~8~~	~~6~~	~~4~~	(10)

4. You should be left with one number. Circle that number.

(9)	~~7~~	~~5~~	~~11~~
~~7~~	(5)	~~3~~	~~9~~
~~13~~	~~11~~	(9)	~~15~~
~~8~~	~~6~~	~~4~~	(10)

5. Add all the circled numbers.

Ask your students to try again, but this time choose different numbers to circle.

What do you notice?

Try using this table

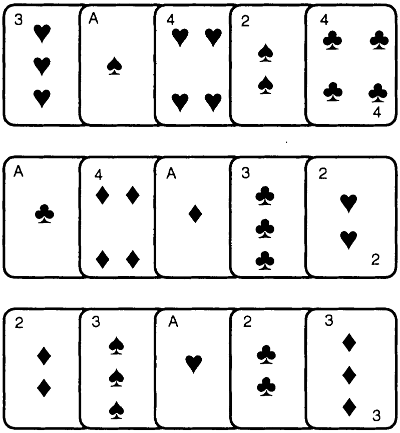
13	**10**	**15**	**12**
6	**3**	**8**	**5**
11	**8**	**13**	**10**
4	**1**	**6**	**3**

What do you notice this time?

For a detailed explanation of how this number novelty works, and for hints on creating your own Tricky Tables, see the Answers and Explanations section.

CRAZY CARDS

1. Deal 15 cards, face up, from an ordinary deck of playing cards, so that 3 rows of 5 are formed.



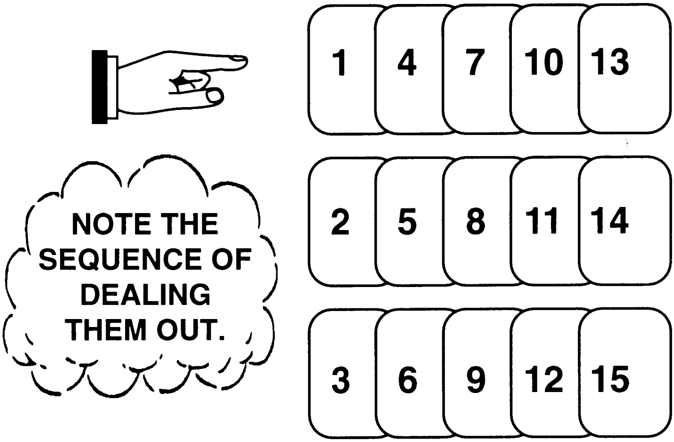
NOTE: It is a good practice to deal the cards out so that each successive card in the row slightly overlaps the previous card. This makes it easy to maintain the order of the cards.

2. Ask a volunteer to choose a card, but not to tell you which it is. (You make like to turn your back while the volunteer shows the rest of the class.)

3. Ask the volunteer to indicate which row contains the chosen card.

4. Collapse each row, ensuring the order of the cards is maintained. Place the row containing the chosen card between the remaining two rows.

5. Deal out the cards again, dealing them out in columns beginning at the top left corner and finishing at the bottom right corner.



6. Ask your volunteer to indicate which row now contains the secret card.

7. Repeat steps 4, 5 and 6 and then collapse the cards again as step 4.

8. Announce that you will reveal the secret card. Count (silently) seven cards from the top of your pack. The secret card will be the eighth card.

Variations

Use 21 cards in 3 rows of 7. You will need to deal the cards three times and then count out 11 cards at the end to find the secret card.

MYSTIFYING MULTIPLICATION

- Ask one of your students to write a 2, 3 or more digit multiplication on the board.

  4362 (Multiplicand)
  x 7746 (Multiplier)

- After the student has written their multiplication on the board, write your own, which contains the same multiplicand on the board.

  4362 (Multiplicand)
  x 2253 (Multiplier)

  The multiplier you choose should be the difference between 9999 and the original multiplier (i.e. 9999 – 7746 = 2253).

- Announce that you will perform both multiplications and add them together in your head, while the rest of the class does the same on a calculator.

The answer may be found by subtracting 1 from your multiplicand (eg 4362) to produce the first first digits of your answer (4361). The last four digits may be found by subtracting this number (4361) from 9999 (9999 – 4361 = 5638). The answer to the problem shown above would be 43615638

Variations:
The same trick can be performed with 2 and 3 digit numbers.

SCRAMBLED DIGITS

Issue the following instructions to one member of your class.

- Choose a six digit number but keep it secret

  458613

- Add up all the digits in the number

  4 + 5 + 8 + 6 +1 + 3 = 27

- Subtract this amount from your original number.

  458613
  –27
  = 458586

- Scramble the digits of the resulting number.

  558684

- Add 25.

  558684
  +25
  558709

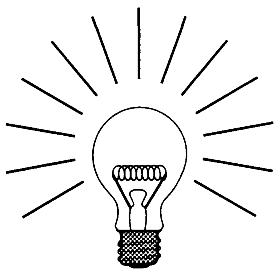
- Cross out any one digit except a zero.

  558 ~~7~~09

- Find the sum of the remaining digits

  5 + 5 + 8 + 0 + 9 = 27

- Ask the student to pass the number they have written to you.



Then tell the student which digit was crossed out.

Answers and Explanations

Page 5. 1089

The same pattern 900 + 180 + 9
is produced due to the restrictions set at the beginning of the puzzle.
The process that leads to the answer of 1089 may be represented algebraically.
If a, b, and c represent the digits in the original number then we get

$$100a + 10b + c - (100c + 10b + a) = 99a - 99c \quad \text{or} \quad 99(a - c)$$

We know that a and c can only represent single digit numbers and that they cannot be equal, due to the restrictions set at the beginning of the puzzle.
Therefore the differences are restricted to multiples of 99 e.g. 198, 297, 396, 495, 594, 693, 792, and 891.
When the numbers are reversed and added it leads to the following combinations:

099	198	297	396	495	
+ 990	+ 891	+ 792	+ 693	+ 594	and vice versa
1089	1089	1089	1089	1089	

Similar reasoning may be applied to 1089 revisited.

1089 x 1 = 1089
1089 x 2 = 2178
1089 x 3 = 3267
1089 x 4 = 4356
1089 x 5 = 5445
1089 x 6 = 6534
1089 x 7 = 7623
1089 x 8 = 8712
1089 x 9 = 9801

There are several patterns. Here are a few.
The thousands and hundreds digits both increase by one while the tens and units digits both decrease by one. If you add the first two digits and the last two digits you always end up with 99. Going down the list of answers, each number comes up again later, but in reverse.

Page 6. 1089 revisited

The final sum always remains the same 109890
There are several patterns:

9 x 1089 = 980
9 x 10989 = 98901
9 x 109989 = 989901
9 x 1099989 = 9899901
9 x 1099989 = 98999901

Note: 2178 is 2 x 1089 therefore 4 x 2178 is really the same as 4 x 2 (or 8) x 1089
There are several patterns.

4 x 2178 = 8712
4 x 21978 = 87912
4 x 219978 = 879912
4 x 2199978 = 8799912
4 x 21999978 = 87999912

Page 7. TAUTONYMS

Following this procedure will always return you to your original three digit number.
Dividing by 7, 11 and then 13 is the same as dividing by 1001. By repeating your three digits to produce a tautonym you have effectively multiplied your 3 digit number by 1001. Dividing by 1001 or (7 x 11 x 13) returns you to your original number.

Page 8. TRIPLE TREAT

You always finish with the number you started with.
This will always happen because dividing by 13, 21 and then 37 is the same as, dividing by 10101. If you multiply any 2 digit number by 10101 you will notice that the two digit number is repeated three times to produce a 6 digit number e.g. 39 x 10101 = 393939. The trick relies on reversing this procedure.
You may like to try writing your own number trick based on multiplying a 2 digit number by 1010101

Page 9. 4 DIGIT MADNESS

6174 gives

$$\begin{array}{r} 7641 \\ -\,1467 \\ \hline 6174 \\ \hline \end{array}$$

in one try

1355 gives 6174 in two steps
5544 gives 6174 in four steps
2333 does not work.

However some numbers with 3 digits the same do work. For example 1777 gives 6174 in six steps. Obviously a number with all 4 digits the same would not work.

Page 10. MULTIPLES OF 9 *(Note: This number novelty is similar to Double Digit Dilemma, p 20)*

1. Yes
2. Yes
3. Yes Simply find the difference between the tens digit and the units digit in your starting number to reveal the multiple of 9 with which you will end up.

Page 11 . CRISS CROSS I

The result will always be 37 because when you cross off the left most digit of the result, which is always 1** and add it on to the units digit you are effectively subtracting 99. Earlier the instruction was to add 62. The difference between 99 and 62 is 37.
If you wish to alter the result simply change the number that you add in the second step. The difference between this number and 99 will be the answer you get at the end of the puzzle.

Page 12 .CRISS CROSS II

This number novelty is very similar to Criss Cross I. The starting numbers have to be between 50 and 100, thereby ensuring that the result will be between 100 and 200 crossing out the left most digit and adding one is the same as subtracting 99.
When the remaining part is subtracted from the original sum it has to leave 99 as the answer.

Page 13. THE MISSING EIGHT

The answer is made up entirely of the number chosen in step 2.
This is because 9 x 12345679 = 111 111 111.
When multiples of 9 are multiplied by 12345679 the answer will consist entirely of the digit corresponding to the multiple of nine that was chosen.

Page 14. SQUARE

The steps in this puzzle essentially describe the binomial expansion. Take for example.

$$
\begin{aligned}
& (8 + 1)^2 \\
= \quad & (8 + 1)(8 + 1) \\
= \quad & (8 \times 8) + (8 \times 1) + (1 \times 8) + (1 \times 1) \\
= \quad & 64 + 8 + 8 + 1
\end{aligned}
$$

Note how the steps in the expansion relate to the instructions that are given Adding the number to itself in **step 1** covers the middle of the expansion (in this case 8+8). **Step 2** is a red herring - the result of zero has no bearing on the outcome. Multiplying by itself covers the first part of the expansion and dividing by itself covers the last part. All that is left to do is add all the parts together.

Page 15. A TRICK FOR SQUARES

This 'trick' is simply another application of the binomial expansion.
Original Number: 23^2

Add one and square: $(23 +1)(23 +1) = 23^2 + 2(23) +1$

- Subtract the smaller number from the larger: $23^2 + 2(23) + 1 - 23^2 \quad = 2(23) + 1$
- Subtract 1: $2(23) + 1 - 1 \quad = 2(23)$
- Halve the number : $\frac{1}{2} \times 2(23) \quad = 23$

So it always returns you to the starting number.

Page 16. PALINDROMES

The difficulty level of this exercise may be altered by changing the numbers. Be careful, however, the size of the numbers you choose may have little bearing to the number of steps it takes to produce a palindrome.
123 (1 step), 632 (1 step), 458 (2 steps), 184 (3 steps), 291 (4 steps), 79 (6 steps) .

Page 17. POINTED PALINDROMES

When you begin with a palindrome you should still end up with a palindrome.

15·4	(1 step),	25·4	(1 step),	16·7	(2 steps),
64·7	(3 steps),	98·3	(2 steps)	21·12	(1 step),
16·61	(2 steps),	17·71	(2 steps),	51·15	(2 steps),
36·63	(5 steps)				

Page 18. DIGIT SHUFFLE

When following this procedure with a 2 digit number the result is always 11
The result for 3 digit numbers is always 222
The result for 4 digit numbers is always 666
Let the two digits be a and b
The numbers formed according to the instructions would be

$$10a + b \text{ and } 10b + a$$

Adding them we get $11a + 11b$

or $11(a + b)$

11 (sum of the original digits)

Therefore, when the total is divided by this sum the answer will always be eleven (11).
Similar reasoning may be applied to three digit numbers. If the three digits are represented by a, b and c the following combinations are formed by the instruction to {write down all the numbers that may be formed by altering the positions of the digits".

a b c	e.g 123
$100a + 10b + c$	123
$100a + 10c + b$	132
$100b + 10a + c$	213
$100b + 10c + a$	231
$100c + 10b + a$	321
$100c + 10a + b$	312

Adding the above we get 222a + 222b + 222c
or $222 (a + b + c)$.
Dividing by the sum of the original digits $(a + b + c)$ leaves 222.
The same reasoning may be applied to the 4 digit numbers.

Page 19. SCRAMBLED DIGITS

The result is always 9, no matter how many digits you start with.

Consider the simplest case of Scrambled Digits which involves a two digit number (ab). The first three instructions lead to the following:
$(10a + b) - (10b + a) = 9a - 9b$ (assuming $a > b$) or $9 (a - b)$
The number formed will always be a multiple of 9, hence the digits will always add to make nine.

Page 20. DOUBLE DIGIT DILEMMA *(Note: This Number Novelty is similar to Scrambled Digits p. 55)*

You always end up with 9 as your answer. In other words a multiple of 9 is always produced.

If the two digits are represented by ab then the steps become

$(10a + b) - (10b + a) = 9a - 9b$ or $9(a - b)$

The difference between the two original digits is $(a - b)$, therefore dividing by $(a - b)$ will always leave 9 as the answer.

Page 21 ***FINDING THE LOST DIGIT*** *(Note: This Number Novelty is similar to Scrambled Digits, p. 55)*

You end up with the missing digit.

Although the following is not a full explanation it does outline the basic principle behind the Number Novelty

digit sum of	$100a + 10b + c$	$= a + b + c$
digit sum of	$10a + c$	$= a + c$
digit sum of difference		$= b$

Page 22. WHO KNOWS.

If we let the digits be represented by a b c and assume $a > b$ and $b > c$
$(100a + 10b + c) - (100c + 10b + a) = 99a - 99c$ or $99(a - c)$
$99(a - c)$ always leads to a multiple of 99
The difference between a and c will always be at least 2 and never more than 8, so the multiples formed will be:

198 297 396 495 594 693 792

In each case the largest digit is the 9. When the largest three digit number is formed the 9 will always end up in the hundreds position. The 9 will always be in the units place when the smallest number is formed.

e.g.

$$\begin{array}{r} 891 \\ -198 \\ \hline 792 \end{array} \quad \text{(This leads to this)} \quad \begin{array}{r} 792 \\ -297 \\ \hline 495 \end{array} \qquad \begin{array}{r} 963 \\ -369 \\ \hline 594 \end{array} \quad \text{(This leads to this)} \quad \begin{array}{r} 954 \\ -459 \\ \hline 495 \end{array}$$

The rest form the same pattern

The result is always 495.

Page 23 TRIPLE DIGIT DIVISION

The answer is always 37.

Let the digit be represented by a

Therefore a three digit number would be $100a + 10a + a$ or $111a$

Adding the digits gives $a + a + a$ or $3a$

$111a \div 3a$ will always leave 37 as your answer.

Page 24 SEEING DOUBLE

Let a, b, and c represent the single digit numbers
The six two digit numbers are formed thus.

$10a + b$
$10a + c$
$10b + a$
$10b + c$
$10c + a$
$10c + b$

Adding the terms we get: $22a + 22b + 22c = 22(a + b + c)$
Dividing by $(a + b + c)$ gives 22 every time.

Page 40. MIND READING TRICK

The cards are produced in such a way that the numbers from 1 to 63 may be formed using various combinations of 1, 2, 4, 8, 16 and 32. These numbers are shown on the top left of each card. As the person indicates which card contains the number, simply note the number on the top left of the card. Add all the numbers as you go. The total will be the person's secret number.

This trick uses base two, similar to computers.

For example $23 = (1 \times 2^4) + (0 \times 2^3) + (1 \times 2^2) + (1 \times 2^1) + (1 \times 2^0)$

$= (1 \times 16) + (0 \times 8) + (1 \times 4) + (1 \times 2) + (1 \times 1)$

Computers would record it as 10111. Thus adding the base two values (top left of each card) gives the value of the chosen number.

Page 42. NUMBER SPELLING

The set of numbers chosen is very special. There is a relationship between the number of letters used to spell each number.

TWO (3) FOUR(4) EIGHT(5) TWELVE (6) FIFTEEN (7)
EIGHTEEN (8) TWENTY ONE (9) TWENTY FOUR (10)

To make the trick work point to any two numbers at random and then point to the others in the order given above. You will note in this case the number of letters and the actual numbers are in order. The third number you should point at is two, then four, eight and so on. If the number chosen was fifteen then the seven numbers will have been pointed at by the time the students have spelled out the seven letters of the word fifteen and you will be pointing at 15.
You may like to vary the numbers.

2	17	11
7	13	
16	25	5

10	19	8
22	16	29
4	20	

The arrangement is not important, but the pointing order *is* important.

Page 43. THRICE DICE

The opposite sides of a die will always add up to seven. Therefore the opposite sides of three dice will always add up to twenty one.
To determine the answer simply look at the number on top of the top die and subtract it from twenty one.

Page 44. WHICH HAND?

If the total is an odd number the five cent coin is in the left hand.
If the total is an even number the five cent coin is in the right hand.

Page 45. WHICH HAND II?

If the answer is odd the odd number of coins is in the left hand.
If the answer is even, then the even number of coins is in the left hand.

Page 46. NUMBER E.S.P.

Here is how to do it.

- The tens digit *will always be* **9**, unless the right hand digit is **0**, in which case the answer will be **0**.
- If the student tells you the right hand digit is **9** then the answer is **99.**
- If the right hand digit is a digit other than **9** or **0** then **subtract** that number from **9** to arrive at the hundreds digit. Using the example on Page 46, if the student told you the last digit was 5, you would know that the tens digit is 9 and the hundreds digit is 4 (because 9 – 5 = 4).

Page 47. THE MISSING NUMBER

To find the missing number reduce the answer by adding the digits until a single digit is produced

e.g. 6 + 6 + 7 = 19
1 + 9 = 10
1 + 0 = 1

Subtract the reduced number from **9** **(e.g. 9 –1 = 8)** to reveal the missing number.

Page 48. GREAT DATES

The answer is always double the current year.

Page 49. DIGIT DETECTION

To find the last digit of the answer, add the two digits given to you and subtract the sum from the next multiple of **9** .

e.g. supplied digits:	**6 and 0**
	6 + 0 = 6
subtract from next multiple of **9:**	**9 – 6 = 3**
Six and zero were supplied so the number is:	**603**

Page 50. TRICKY TABLE

The students should always find that their circled numbers add up to 33. This is because the original table was produced from an addition table.

e.g.

+	**5**	**3**	**1**	**7**
4	**9**	**7**	**5**	**11**
2	**7**	**5**	**3**	**9**
8	**13**	**11**	**9**	**15**
3	**8**	**6**	**4**	**10**

To make your own "magic table" simply start with an addition table of your own choice.

e.g.

+	**4**	**1**	**6**	**3**
9	**13**	**10**	**15**	**12**
2	**6**	**3**	**8**	**5**
7	**11**	**8**	**13**	**10**
0	**4**	**1**	**6**	**3**

Remove the top row and left column to leave a 4 x 4 table.

You can determine what the 'secret total' will be by simply adding the top row and left column of your original table.

(4 + 1 + 6 + 3) + (9 + 2 + 7 + 0) = 32

Variations

1. Create a larger table by altering the number of entries in the top row and left column. Note there should always be the same number of rows and columns, so that you are left with a square table.
2. Create a multiplication table. An example of a multiplication table is shown on the next page. Here the students would have to multiply their final circled numbers.
3. Lower secondary teachers might like to create a subtraction table. This would provide good practice in adding negative numbers.

MULTIPLICATION TABLE

ORIGINAL TABLE

x	1	3	5	7
2	2	6	10	14
4	4	12	20	28
1	1	3	5	7
6	6	18	30	42

The secret number is:

(1 x 3 x 5 x 7) x (2 x 4 x 1 x 6) = 5040.

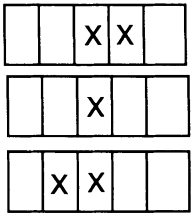
The steps are the same as in the previous example, except that when all the circled numbers have been found, they are multiplied instead of being added.

~~2~~	~~6~~	(10)	~~14~~
~~4~~	~~12~~	~~20~~	(28)
~~1~~	(3)	~~5~~	~~7~~
(6)	~~18~~	~~30~~	~~42~~

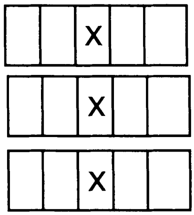
6 x 3 x 10 x 28 = 5040

Page 52. CRAZY CARDS

Dealing the cards in the sequence outlined ensures that the chosen card always ends up in the centre of the middle row. the chosen card is forced to the centre by the order in which they are dealt out. Being in the middle set of 5, after the first deal, the card is one of those marked.



After the second deal, the card is one of:



When placed in the middle row, the chosen card is the middle one (i.e. the eighth card).

Page 54. MYSTIFYING MULTIPLICATION

To make this trick work you must subtract your student's multiplier from 9999

e.g.

$$\begin{array}{r} 9999 \\ -7746 \\ 2253 \end{array}$$

and use the result as your multiplier. In other words the sum of the two multiplications will be the same as multiplying the original multiplicand by **9999**. All you have to do to arrive at the answer is to subtract **1** from your multiplicand to find the first set of digits in your answer and then subtract this number from **9999** to find the last 4 digits in the answer.

The answer for **4362 x 7746** and **4362 x 2253** is **43615638**

Mathematically, $n \times 9999 = (n \times 10000 - n)$

$$= (n - 1) \times 10000 + [9999 - (n-1)]$$

Variation: The same principle works for 2 and 3 digit numbers.

Page 55. SCRAMBLED DIGITS (NOTE: *This Number Novelty is similar to Finding The Lost Digit, p21)*

Subtracting the sum of the digits in the original number leaves a number whose digits add up to an exact multiple of **9**. Adding **25** produces a sum that is **7** more, an exact multiple of **9**. Therefore to find the missing digit simply subtract **7** from the final digit sum and then subtract this amount from the next highest multiple of **9**.

Variation: Instead of adding **25** you may use any other number. When working out the missing digit, instead of subtracting **7** from the final digit sum you need to subtract the digit sum of the number you added earlier (e.g. if you added **32** you would subtract **5 (3 + 2)** from the digit sum and then subtract that amount from the next highest multiple of **9.**

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