A Guide to...

MATHEMATICAL VOCABULARY

For Comprehending and Solving Word Problems

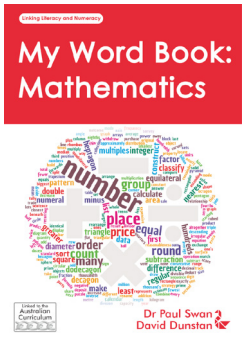
About: Vocabulary

Vocabulary knowledge is a predictor of comprehension ability. Hirsch and Nation (1992) indicate that students need to know between 90 to 95 % of the words in a text in order to comprehend the text*.

There is no "one method" for teaching mathematics vocabulary. In this guide we present a collection of routines that may be used over and over which we call a 'menu'. Once you (the teacher) become familiar with the routines you will find that you can easily differentiate the activities to meet the needs of your class.

*It should be noted that they were discussing general reading and not the technical type of reading required in mathematics and science where the text is dense. If anything, the requirement for mathematics may be even higher.

Starting Point: A Word List



Purpose: To begin, you need a word list.

Material: My Word Book Mathematics contains appropriate word lists by year level

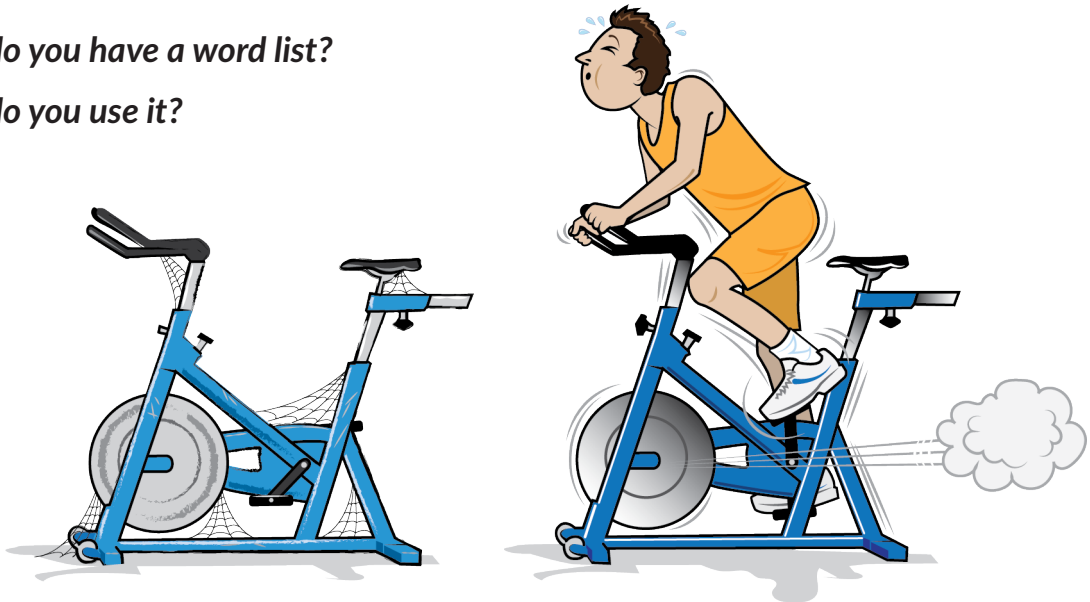
You're not required to use our word list to use this guide.

See also: A free set of NAPLAN word flash cards for Year 3 can be found at www.drpaulswan.com.au/maths-topics.

NAPLAN word flash cards for Years 5 & 7 are available to buy at www.drpaulswan.com.au/shop.

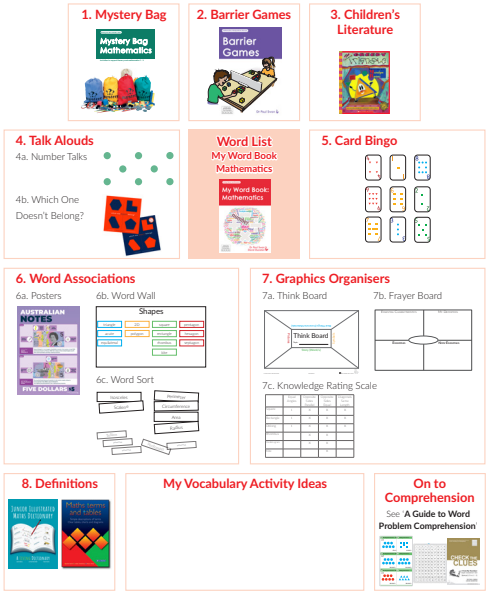
Question 1: do you have a word list?

Question 2: do you use it?



Having an exercise bike and using one are two different things. This guide is about ***using*** the word list.

Using this Guide



This guide is set out around the idea of a 'menu'. You pick items off of a menu based on your needs. The menu is presented in a general order, starting with simply exposing the children to the words, moving through using the words and ending with defining the words.

All items on the menu draw on the same 'ingredients' - the word list.

Your students might know most of the vocabulary related to number, so you may want to use the **Knowledge Rating Scale** to assess and check, proceeding as needed. However you may know that their geometry vocabulary is lacking, so you could begin with a **Barrier Game** or a **Children's book**.

It is recommended that you use a combination of several activities to build the students' mathematical vocabulary.

Teaching Vocabulary

Beck, McKeown, and Kucan (2002) outline three tiers of vocabulary that need to be taught.

Tier One

Everyday words that students generally come to school with or words they learn in the early years#.

Tier Two

Words that are more academic but not specific to any one discipline. Words like analyse or evaluate would qualify as a Tier Two words. Tier Two words tend to receive less attention because they do not belong to any one discipline.

Tier Three

Words that are content specific words, such as 'hypotenuse', that Marzano (2004) would call academic vocabulary. Tier three words are typically used only in the school or University setting, hence why they are referred to as academic vocabulary.

Direct Methods

Marzano (2004) suggests six steps when learning vocabulary:

1. **The teacher provides an explanation or example/description of the word or phrase in simple terms.**

The Junior Illustrated Dictionary provides some support. Note the definitions used here are less formal than Maths Terms and Tables

2. **Students restate the description in their own words.**

Students using the words and describe them

#Students whose first language is not English could have gaps in their knowledge here and will need assistance.

3. **Students represent the term in a drawing or some form of representation.**

Students can draw pictures, link words to symbols or even use gestures to describe what a term means.

4. **Teacher expands and refines the use of the word.**

See activities on the menu.

5. **Find opportunities for students to discuss words and phrases with peers.**

This means spaced practice using menu activities. See specifically barrier games, talk alouds and posters / word walls / word sorts.

6. **Strengthen word knowledge through various activities.**

This means spaced practice using menu activities.

Notice how Marzano builds toward a formal definition rather than starting with it.

Stahl and Fairbanks (1986) recommend that students be exposed to words at least seven times over spaced intervals for retention to occur.

Summary

- Vocabulary is an essential element of solving word problems.
- In mathematics, what might be thought of as Tier One words can prove to be tricky. Consider the use of the words, 'and' and 'or'. When applied to Venn diagrams these words change meaning dramatically compared to their everyday use. For example, consider the phrases '25% of ten dollars' and '25% off ten dollars'.
- Marzano (2004) found that teaching academic vocabulary could positively influence standardised test scores.
- Students need to be exposed to words at least seven times initially and then continually over a period of time.

References

Beck, I. L., McKeown, M. G., & Kucan, L. (2002). ***Bringing words to life: Robust vocabulary instruction.*** New York. Guilford Press.

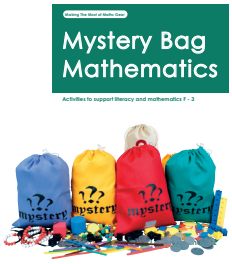
Hirsh, D., & Nation, P. (1992). ***What vocabulary size is needed to read unsimplified texts for pleasure?*** Reading in a Foreign Language, 8, 689–696.

Marzano, R. J. (2004). ***Building background knowledge for academic achievement: Research on what works in schools.*** Alexandria, VA: ASCD.

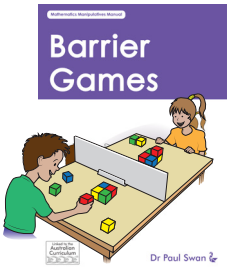
Stahl, S. A., & Fairbanks, M. M. (1986). ***The effects of vocabulary instruction: A model-based meta-analysis.*** Review of Educational Research, 56(1), 72-110.

Swan, P., & Dunstan, D. (2018). ***My Word Book: Mathematics***. Perth: A-Z Type.

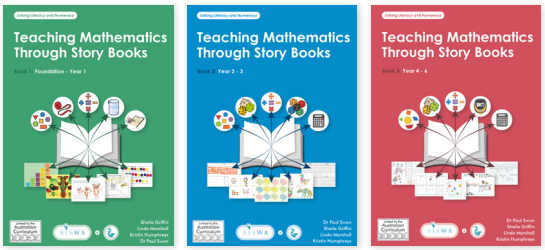
1. Mystery Bag



2. Barrier Games

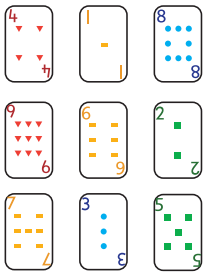


3. Children's Literature

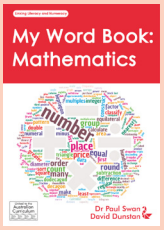


Teaching Mathematics Through Story Books Series

4. Card Bingo



Word List
My Word Book Mathematics



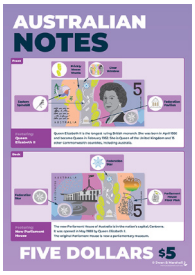
5. Talk Alouds

5a. Number Talks

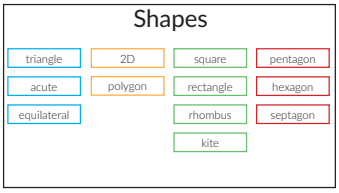
5b. Which One Doesn't Belong?

6. Word Associations

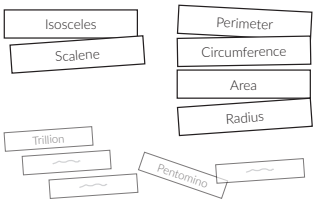
6a. Posters



6b. Word Wall

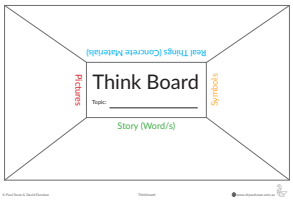


6c. Word Sort

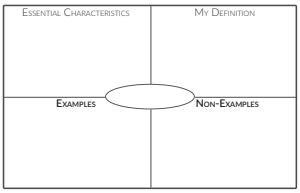


7. Graphics Organisers

7a. Think Board



7b. Frayer Board



7c. Knowledge Rating Scale

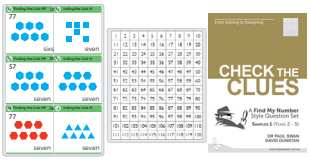
	Equal Angles	Opposite Sides Parallel	Opposite Sides Equal	Diagonals Same Length
Square	X	X	X	X
Rectangle	X	X	X	X
Oblong	X	X	X	X
Rhombus		X	X	
Parallelogram		X	X	
Kite			X	

8. Definitions

My Vocabulary Activity Ideas

On to Comprehension

See '**A Guide to Word Problem Comprehension**'



Example Word List - NAPLAN Word List Year 3

These aren't all the words that are in Year 3, but this list contains just the subset of Year 3 words that have appeared in NAPLAN tests spanning close to a decade. Taken from My Word Book: Mathematics

Number and Algebra

add(ed) / subtract(ed)	in total
altogether	left
amount	more
arrange	more than
arrow points	most / least
buy / sell, sold	next number
change	not
cheap, cheapest	number
close, closest (to)	number line
clues	number sentence
collect, collections	one more / one less
cost	ones place
counting down	only
days later	pattern
different, difference	place (ones, tens ...)
digit	problem
each	quarter (turn)
equal	removed
estimate, best estimate	repeat (pattern)
exactly	row / column
extra	score
fewer	second from the
four	second oldest
greatest/ least number	shortest / tallest
groups of	solve
half way, half / twice	some
how far	tens
how many	three times
hundreds place	whole

Statistics and Probability

certain	less, least
chance	likely / unlikely / equally likely
chart	picture graph
data	possible / impossible
fair	spins / spinner
favourite	table
fewer	tally
flip	times
graph	tossed a coin
heads / tails	

Measurement and Geometry

(more / half) full	least
3D objects	left / right
area	length, total length
around	light / heavy
balanced, balance (scale)	litre
between	located
block	map
cell	mass
centimetres	measure, tape measure
cents	metres
circle	missing
circular	model
clock, clockwise, anticlockwise	months (names)
closed	net
container	pattern
cube	position, positioned
cup	prism
cylinder	rectangular (prism / pyramid)
different/ same	right hand
direction	same size
East / West / North / South	scales
edges / faces / vertices	seasons (names of)
floor plan / plans	shape (shape names)
fold / unfold	shorter / longer / taller
front / side / top view	sphere
furthest away from	square (based pyramid)
gram	start / end / finish
grid paper	symmetry / symmetrical
half / quarter past (to)	tile
half as wide	time
heaviest/ lightest	top / bottom
heights	triangle (based prism / pyramid)
hours, minutes, seconds	turn, turning
joining	view
kilogram	weighs / weight
kilometres	
largest	
layer/ layers	

Vocabulary Development Across the School

To ensure the coherent development of vocabulary across the school consider the chart below.

As a school, choose menu options that will receive a stronger focus at each year level.

F	1	2	3	4	5	6	7-10
Mystery Bag							
Barrier Games							
Children's Literature							
Card Bingo							
Talk Alouds (Number Talks, Which One Doesn't Belong? etc.)							
Word Associations (Posters, Word Walls, Word Sorts)							
Graphics Organisers (Think Board, Frayer Board, Knowledge Rating Scale)							
Definitions							

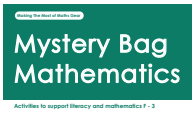
Utilising a "Menu" approach.

Just like a menu at a restaurant, you wouldn't choose every single option at once.

Select one or two at a time ***based on school/class/student needs.***

MENU OPTIONS & PURPOSE

Mystery Bag



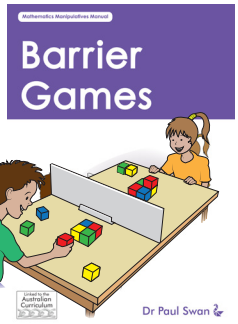
Purpose: Practise descriptive language.

Material: An opaque bag or box and items to put inside.

See also: Mystery Bag Mathematics by Dr Paul Swan & Linda Marshall

1.

Barrier Games



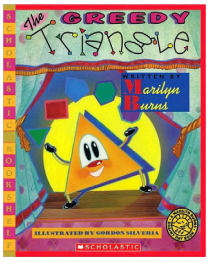
Purpose: Builds expressive and receptive language (talking and listening) - ideal for developing prepositions ("next to", "above", "below"). Great for improving geometry related vocabulary.

Materials: A physical barrier and a game to play (e.g. Battleship).

See also: Barrier Games by Dr Paul Swan

2.

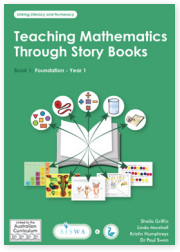
Children's Literature



Purpose: Exposure to mathematical language in a narrative.

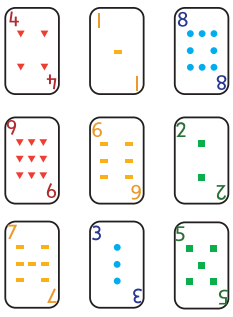
Material: Children's books, *Teaching Maths through Story Books* titles

See also: List of Children's Literature.



3.

Card Bingo



Purpose: Appropriate for all year levels, this is a simple routine that can be used as a vocabulary and basic facts warm up (number vocabulary only).

Material: Cards in the appendix items or School Friendly Cards

See also: Dr Paul Swan's A Guide to Warm Ups

4.

Talk Alouds

Number Talks

Purpose: Encourage students to use mathematical vocabulary to describe situations.

5a.

Which One Doesn't Belong?

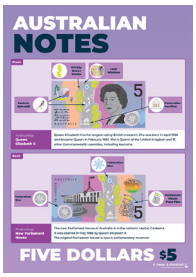
Purpose: Students use their vocabulary to explain why in each case an item doesn't belong.

See also: www.wodb.ca

5b.

Word Associations

Posters

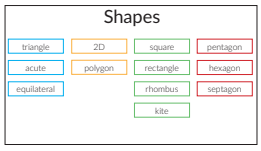


Purpose: Classroom Posters highlight either individual words or words associated with a topic (e.g. shape words). Posters are best used as references to current topics - otherwise they risk becoming glorified wallpaper.

See also: Dr Paul Swan Money Posters

6a.

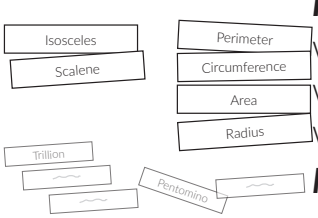
Word Wall



Purpose: Students brainstorm the words to go up on the wall on a given topic. Word Walls are dynamic - they grow as students add more words as their knowledge of the topic grows. Word Walls passively remind students of the words related to the current topic.

6b.

Word Sort



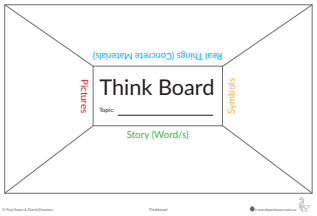
Purpose: Take a set of words and classify them in different ways. This requires more than just a knowledge of the word - the student needs to know how it relates to other words.

Material: Word Sort Cards

6c.

Graphic Organisers

Think Board

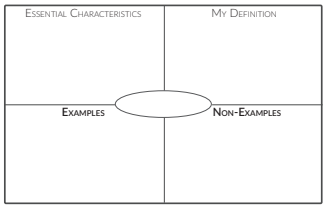


Purpose: The Think Board helps link physical materials, pictures, words and symbols - helping to build word problem comprehension.

Material: Think Board & Think Board Cards

7a.

Frayer Board (Four Square)



Purpose: The Frayer Board links pictures, words and definitions. Typically used with older students.

Material: Frayer Board

7b.

Knowledge Rating Scale

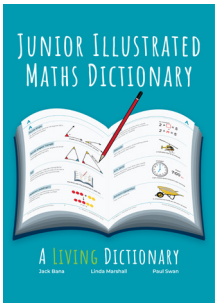
	Equal Angles	Opposite Sides Parallel	Opposite Sides Equal	Diagonals Same Length
Square	X	X	X	X
Rectangle	X	X	X	X
Oblong	X	X	X	X
Rhombus		X	X	
Parallelogram		X	X	
Kite			X	

Purpose: This is a great tool to use to assess student knowledge. Put a Knowledge Rating Scale sheet in between a group of four students and ask about a set of words. Students will mark from 'never heard of it' to 'I can describe/define it'

Material: Knowledge Rating Scale Sheet

7c.

Definitions



Purpose: According to Marzano (2004) formal definitions develop over time. Definitions are concise explanations of what a word means, and as such often require broad vocabulary knowledge and other definitions.

See also: Junior Illustrated Maths Dictionary by Dr Jack Bana, Dr Paul Swan & Linda Marshall. This illustrated dictionary is designed to link graphics and words.

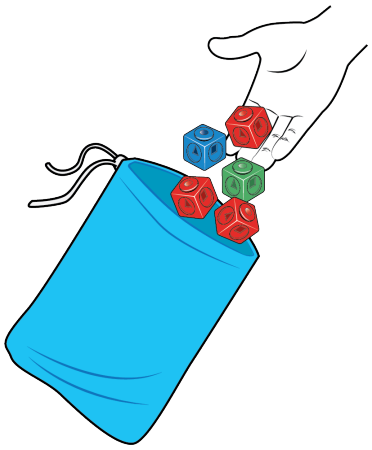
Maths Terms & Tables (Bana, Swan, Marshall) uses more formal definitions and covers harder terms.

8.

1. Mystery Bag Activities

Background

Objects may be placed inside the 'Mystery Bag' and children can then reach into the bag and feel the object inside the bag. This tactile experience leverages the use of an extra sense (touch). Here are some different examples:



The Long and Short of it

Place three coloured rods or three sticks of different lengths into the Mystery Bag and ask the children to pull the rods out of the bag, one at a time, from shortest to longest.

Odd Objects

Place some 3D objects inside the Mystery Bag and ask the children to draw out objects according to certain criteria e.g. prism, pyramid, "has an apex", "will roll"...

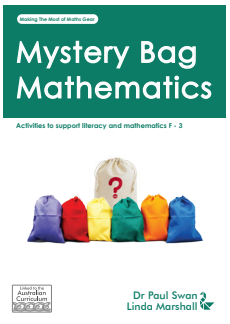
Coin Experiences

Place one of each coin into the Mystery Bag.

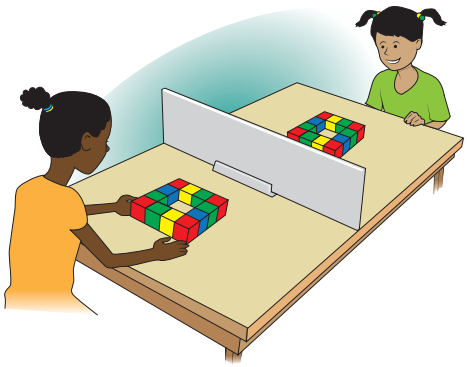
Children can be asked to pick a particular coin out of the bag. For example, the children might be asked to pull out:

- the smallest coin
- the fifty-cent piece
- coins in order of size - smallest to largest
- two coins of roughly the same size
- two coins with the largest size difference
- coins in order of value

More comprehensive use of the Mystery Bag can be found in the title Mystery Bag Mathematics available from www.drpaulswan.com.au



2. Barrier Games



Barrier games (think Battleship) involve giving and receiving instructions. Generally the instructions are in oral form but they may be written down.

Barrier Games are particularly good for geometry and location vocabulary.

Teachers can differentiate the difficulty level of Barrier Games by;

- changing the objects used.
- changing the vocabulary to be used:
  - by restricting/increasing the vocabulary,
  - by providing vocabulary cards,

What will Students Learn?

- receptive language: to listen, understand and interpret what is said
- expressive language: to explain in a way that makes sense
- to give explicit instructions
- to ask clarifying questions
- mathematical vocabulary

Activity: Block Building

Have two or more players sitting either side of a physical barrier. One player (the instructor) builds something and explains to the player on the other side of the barrier (the listener) how to build a replica. When the barrier is removed the original model is compared with the one constructed by the listener. The players thus receive instant feedback.

Assessment of Barrier Games

Students receive immediate formative assessment when they lift the barrier.

Teachers can observe two players while they are engaged in a Barrier Game.

Students can be asked to provide another player with written instructions, which in turn may be assessed to see if the instructions match the finished product.

More comprehensive use of the Mystery Bag can be found in the title Barrier Games available from www.drpaulswan.com.au

3. Children's Literature that Incorporates a Mathematics Theme

Developed by Linda Marshall and Paul Swan to assist teachers looking to incorporate childrens literature into mathematics lessons. These books have been grouped under various themes in alphabetical order. Brief comments have been made for each publication; and links made to the Australian Curriculum: Mathematics.

Number and Counting:

Author(s)	Reference	
Allum, M., & Watson, J.	(2005). **How Many Peas In A Pod?** Surry Hills, NSW: Little Hare Books.	ACMNA001
A counting book from 1 – 12 using flaps to be lifted to reveal the answers to each question.		
Base, G.	(2006). **Uno's Garden**. Australia: Penguin Group.	ACMNA012
A beautiful counting book; counting down, square numbers, doubling (balance).		
Base, G.	(1995). **The Waterhole**. Camberwell, Victoria: Puffin Penguin.	ACMNA012
Counting book based on a diminishing water hole; rain at the end!		
Birch, D.	(1988). **The King's Chessboard**. New York: Puffin Books.	ACMNA018
Number, pre-algebra, exponential growth.		
Burningham, J.	(1980). **The Shopping Basket**. London: Red Fox Books.	ACMNA012
Steven goes shopping and buys 6 eggs, 5 bananas, etc. On the way home, animals wanting his goods meet him.		
Carle, E.	(2005). **10 Little Rubber Ducks**. London: HarperCollins Publishers Ltd.	Ordinal number
A beautifully illustrated book that looks at 10 ducks washed from a boat. Uses 1st, 2nd, 3rd, etc. to tell the story of each duck.		
Carle, E.	(1995). **Rooster's Off To See the World**. London: Puffin Books.	ACMNA012
Rooster decides to travel and meets up with two cats, three frogs, etc. Later they return to their homes, counting back down from 5 fish to one Rooster.		
Cave, K.	(2002). **One Child One Seed: A South African Counting Book**. Great Britain: Frances Lincoln Ltd.	ACMNA012
A multicultural look at counting.		
Dale, P.	(1988). **Ten in the Bed**. London: Walker Books.	ACMNA012
Counts down from 10 according to the rhyme.		
Fromental, J., & Jolivet, J.	(2006). **365 Penguins**. [English translation] JY: Harry N. Abrams, Inc.	ACMNA018
A fantastic number book with patterns for days throughout the year.		
Hutchins, P.	(2000). **Ten Red Apples**. London: Red Fox.	ACMNA012
Counts down from 10 using repetitive language that the children can join in with,		
Jandl, E., & Junge, N.	(2003). **Next Please**. London: Red Fox.	ACMNA012
Wonderful counting backward from 5 to 0 in a to 'doctor's' waiting room.		
Milborne, A. & Riglietti, S.	(2007). **How Big is a Million?** London: Usborne Publishing Ltd.	ACMNA072
A journey of discovery to find a million.		
Oliver, N.	(2003). **The Very Blue Thingamajig**. Sydney: Scholastic.	ACMNA001
A delightful and unusual counting book that would really appeal to children's imaginations.		

Parish, S.	(1998). **123 of Australian Wildlife**. Queensland: Steve Parish Publishing Pty Ltd	ACMNA001
A beautiful Australian book featuring colour photos of native Australian wildlife.		
Parker, V., & Bolam, E.	(2002). **Bearum Scarum**. London: Hodder Children's Books.	ACMNA012
A book where the hunter becomes the hunted. An amusing counting back from ten book.		
Pinczes, E.	(1993). **One Hundred Hungry Ants**. Boston: Houghton Mifflin.	ACMNA031
Number, factors, division		
Puttock, S.	(2006). **Don't Count Your Chickens**. London: Macmillan Children' Books.	ACMNA012
Doubling (counting by twos).		
Robinson, T.	(2015). **Fibonacci Zoo**. Abordale publishing	
Explores the Fibanacci number pattern.		
Root, P.	(1999). **One duck stuck**. London: Walker Books.	ACMNA001
Counts up to 10 using beautiful descriptive language.		
Ross, T.	(2002). **Centipede's 100 shoes**. London: Andersen Press Ltd.	ACMNA014
An amusing story with addition and subtraction using shoes and socks.		
Sayre, A., & Sayre, J.	(2003). **One is a Snail Ten is a Crab**. London: Walker Books.	ACMNA015
A counting book to 100, using feet. Shows different combinations for many numbers e.g., 30 is 3 crabs or 10 people and a crab.		
Schwartz, D.	(1989). **If You Made a Million**. New York: Mulberry Books	ACMNA072
Number, large numbers, percentages, interest.		
Schwartz, D.	(1985). **How Much is a Million?** NY: Scholastic Inc.	ACMNA072
One million, large numbers.		
Straw, W.	(2001). **5 Little Ducks**. Melbourne: Borghesi and Adam Publishers.	ACMNA012
Counts down from 5 according to the popular song. Big, clear illustrations.		
Trinka, R., & Argent, K.	(1999). **One Woolly Wombat**. SA: Omnibus Books.	ACMNA001
Basic counting book.		
Wells, R.	(2000). **Can You Count to a Googol?** Illinois: Albert Whitman & Company.	ACMNA072
Number, counting to large numbers		
Wood, A., & Wood, B.	(2004). **Ten Little Fish**. New York: Scholastic Inc.	ACMNA012
Counts colourful fish from 10 to 1 and back. Leaves each answer to the following page.		

Operations and Computation:

Anno, M., & Anno, M.	(1983). **Anno's Mysterious Multiplying Jar**. New York: Putnam & Grosser Group.	ACMNA076
A beautifully illustrated book that covers the topic of factorials ie 5 x 4 x 3 x 2 x 1.		
Calvert, P.	(2006). **Multiplying Menace: The Revenge of Rumpelstiltskin**. MA: Charlesbridge.	ACMNA076
Number, multiplying, division, fractions.		
Clement, R.	(1990). **Counting on Frank**. North Ryde, NSW: Bluegum.	Reasoning
A very popular book that involves estimation and encourages children to work mathematically.		

Dodds, D. A., & Mitchell, T.	(2000). **The Great Divide: A Mathematical Marathon**. London: Walker.	ACMNA056
As the name implies this book involves the division concept – especially halving.		
Hutchins, P.	(1986). **The Doorbell Rang**. London: Penguin.	ACMNA056
This story involves sharing cookies among a group of children.		
Neuschwander, C.	(2007). **Patterns in Peru: An Adventure in Patterning**. New York: Henry Holt.	
Pre-algebra		
Neuschwander, C.	(1998). **Amanda Bean's Amazing Dream: A Mathematical Story**. New York: Scholastic.	
Multiplication, repeated addition, arrays		
Pinczes, E.	(1995). **A Remainder of One**. Boston: Houghton Mifflin.	
Number, division, remainders		
Tang, G.	(2003). **Math Appeal: Mind Stretching Math Riddles**. New York: Scholastic.	ACMNA031
This book follows on from The Grapes of Math and is aimed at children moving from addition to multiplication.		
Tang, G.	(2003). **Math-terpieces: The Art of Problem-solving**. New York: Scholastic.	ACMNA030
Uses well-known works of art to motivate children to find different ways to add.		
Tang, G.	(2002). **The Best of Times: Math Strategies that Multiply**. New York: Scholastic.	ACMNA056
Uses problem rhymes to show better ways to multiply numbers from zero to ten,		
Tang, G.	(2001). **The Grapes of Math: Mind-stretching Math Riddles**. New York: Scholastic.	
Uses problem rhymes to look at simple computations in a different way, looking at some interesting strategies.		

Geometry:

Author(s)		
Burns, M,	(nd). **The Greedy Triangle**. New York: Scholastic Inc.	ACMMG022
Geometry: 2D Shape		ACMMG043 ACMMG063
Haas, K.	(2014). **The Shape Family Babies**. Abordale Publishing.	ACMMG022
A book about shapes and angles		ACMMG043 ACMMG063
Hennessy, B., & Joyce, P.	(2004). **The Once Upon a Time Map Book**. London: Walker Books.	ACMMG065
A wonderful trip to 6 different story lands with maps, coordinates, routes, hidden objects and points of interest.		ACMMG090
Hutchins, Pat.	(1968). **Rosie's Walk**. London: Red Fox.	ACMMG010
This book can be used to encourage the language of direction.		ACMMG023

Measurement:

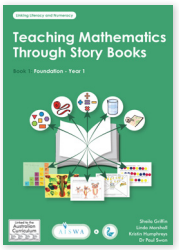
Author(s)		
Alborough, J.	(1997). **Watch out! Big Bro's coming**. London: Walker Books.	ACMMG037
A humorous tale about a mouse's big brother who appears to grow according to who describes him.		
* Allen, P.	(1994). **Alexander's Outing**. Victoria: Puffin Books.	ACMMG006
Story about a duck falling in a hole; possibilities for volume.		
Allen, P.	(1980). **Mr Archimedes' Bath**. Sydney: William Collins.	ACMMG037 ACMMG290
Mr Archimedes notices that when he and his animal friends get in and out of the bath, the water lever changes.		
Anolt, L.	(2001). **Knee High Nigel**. London: Walker Books.	ACMMG019 ACMMG037
The story of five giants, one of whom, though still a giant, is considerably smaller than the others. They argue over the building of castles, and go their separate ways with unsuccessful results.		
Billington, J., & Smee, N.	(1999). **Six Feet Long and Three Feet Wide**. London: Walker Books.	ACMMG061
This story illustrates the need for standard units.		
Briggs, Raymond.	(1970). **Jim and the Beanstalk**. London: Penguin.	ACMMG084
This book can be used to give an intuitive idea of scale.		
Burns, M,	(1997). **Spaghetti and Meatballs for all!** A mathematical story. New York: Scholastic Inc.	ACMMG109
Measurement, Area and perimeter		
Carle, E.	(1977). **The Bad Tempered Ladybird**. London: Puffin Books.	ACMMG020
A ladybird works its way through different times of the day. Shows the time on analogue clocks.		
Carle, E.	(1970). **The Very Hungry Caterpillar**. London: Penguin.	ACMMG008
A caterpillar eats its way through the week.		
Dunbar, James.	(2004). **Tick-Tock**. London: Franklin Watts.	ACMMG021 ACMMG085
This book provides an opportunity to talk about units of time (seconds, minutes, hours, days, weeks, months, years, decades, seasons), time passed and time still to come.		
Gliori, D.	(2013). **What's The Time Mr Wolf**. Bloomsbury Publications	
Focusses on time throughout the day – with a mix of poplar nursery rhymes and stories.		
Hawkins, C.	(2003). **Mr Wolf's Week**. London: Egmont Books Ltd.	ACMMG008
The cycle of the days of the week shown through Mr Wolf's amusing antics		
Hindley, J., & Chamberlain, M	(1993). **A Piece of String is a Wonderful Thing**. London: Walker	ACMMG139
Traces the idea that in different eras of human history string would have been a useful tool. Times lines could be made from the information. So, how long is a piece of string? Twice as long as half its length.		
* Hughes, N.	(2004). **Colossal Machines**. Mascot, Australia: Koala Books.	ACMMG061
Comparison of large machines to dinosaurs;; length and mass.		
Hutchins, P.	(1974). **Clocks and More Clocks**. London: Penguin.	ACMMG086
As the name implies this book involves problem solving and time.		
Hutchins, P.	(1997). **Shrinking Mouse**. London: Red Fox.	ACMMG037
Deals with perspective – things look smaller from a distance.		
Matthews, P., & McLean, A.	(2002). **A Year on our Farm**. Norwood, SA: Omnibus Books.	ACMMG040
At last, a book illustrating months of the year and the 4 seasons in a distinctly Australian setting.		
Myller, R.	(1962). **How Big is a Foot?** New York: Dell Yearling.	ACMMG061
This story illustrates the need for standard units.		
Looks at big, bigger, biggest; small, smaller, smallest; and growing.		

Micklethwait, L.	(2004). **I Spy Shapes in Art**. London: Harper Collins.	ACMMG009 ACMMG022 ACMMG042
A wonderful book that features real pieces of art. Uses paintings from Matisse to Warhol to look at geometric shapes.		
* Mitchell, A.	(1998). **Twice my Size**. London: Bloomsbury. [Republished Igloo 2007]	ACMMG242
Comparative size of objects starting with a lady bird; roughly doubling each time,		
Neuschwander, C.	(2001). S**ir Cumference and the Great Knight of Angleland**. MA: Charlesbridge.	
Geometry, angle		
Neuschwander, C.	(1999). **Sir Cumference and the Dragon of Pi**. MA: Charlesbridge.	
Geometry, discovering Pi		
Neuschwander, C.	(2003). **Sir Cumference and Sword in the Cone**. MA: Charlesbridge.	
Geometry 3D shapes		
Neuschwander, C.	(1997). **Sir Cumference and the First Round Table**. MA: Charlesbridge.	
Geometry, 2D shape		
Neuschwander, C.	(2005). **Mummy Math**: An adventure in geometry. New York: Square Fish	
Geometry 3D shape		
Schwartz, D.M.	(1999). **If you Hopped like a Frog**. NY: Scholastic Press	
Units of measurement; activities in the back.		
* Slater, T.	(1996). **Just a Minute!** UAS: Scholastic Inc.	ACMMG062
What is a minute?		
Wells, R E.	(1995). **Is the Blue Whale the Biggest Thing There is?** London: Watts Books.	ACMMG037
A book of comparisons. The sizes of various things are compared.		
Whybrow, I., & Reynolds, A.	(2002). **Harry and the Dinosaur Have a Very Busy Day**. Mascot, NSW: Koala Books	ACMMG009 ACMMG022
This book looks at 5 different shapes, and could lead to discussion about different shapes in the classroom, in the playground, and in the environment generally		

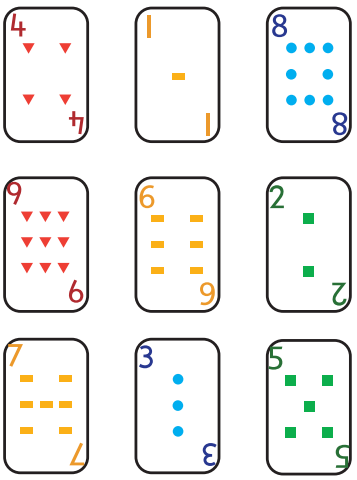
Probability and Statistics:

Author(s)	Reference	
* Anno, M., & Mori, T.	(1985). **Anno's Three Little Pigs**. London: the Bodley Head Ltd.	Probability
Permutations and combinations specifically in story,		
Einhorn, E.	(2008). **A Very Improbable Story**. MA: Charlesbridge.	ACMSP092 ACMSP093
Probability		
Rodda, E.	(1986). **Pigs Might Fly**. Sydney: Harper Collins.	ACMSP092
A novel designed for middle to upper primary aged children. Contains wonderful language associated with chance.		
* Smith, D.J.	(2006). **If the World Were a Village**. NSW: Allen & Unwin.	ACMSP117 ACMSP144
Comparative data about the world and its people.		

The Mathematics Association of Victoria (https://shop.mav.vic.edu.au/products) maintains a wide range of Children's Literature, which you can browse by Year Level and topic. To learn how to better use some of these children's books, see our titles in the Teaching Mathematics Through Story Books series.



4. Card Bingo



Purpose

Students will focus on the language of mathematics while learning the basic number facts.

Materials

The cards 1 - 9 from a deck of School Friendly Cards (or the cards on the following page) per player. That is, for a class of 32, you would need eight decks/8 sets of printed cards 1-9.

Organisation

A game for the whole class and one Caller.

Aim

To be the first player to turn over three cards in a row, column or diagonal.

Rules

1. Players arrange their nine cards into a 3 x 3 bingo board. The cards should be in mixed order.
2. An operation and appropriate terminology are chosen. For example, subtract one, might be called the 'Take One' game, however on another occasion it might be referred to as the 'Subtract One' game; on another occasion the 'Minus One' game. The 'Number Before' game will give the same result, however, the concept behind this game is different as it involves the idea of position.
3. The Caller has a set of cards numbered 1 - 9 and shuffles these to ensure that the cards are randomly chosen.
4. If playing the 'Take One' game, the Caller, in the first instance, the teacher, would mentally add one to the card that he/she turns over. For example, if the Caller turned over a 7, he/she would call out 'eight'.
5. The players would then hear 'eight' and think that they have to take one away from eight and turn over the card with '7' on it.
6. Play continues until one player or several players turn over three cards in a row, column or diagonal as per the normal rules for Bingo.
7. The winners must then hold up their cards and compare them with the caller's set of cards to verify they have the correct answers.

Variations

Change the language, eg play the 'One Fewer Than' game. Teachers can watch to see whether children who could play the 'Take One' game struggle with this new game, perhaps indicating a difficulty with the language.

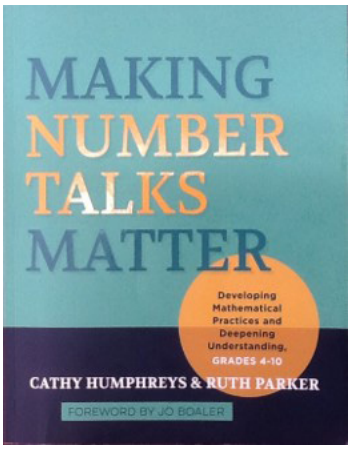
Extend to much harder operations that are linked. For example, the 'Halving' game, the 'Dividing by Two' game, the 'Fifty per cent' game and the 'Multiply by 0·5' game. Students can be asked what they notice about each of these games.

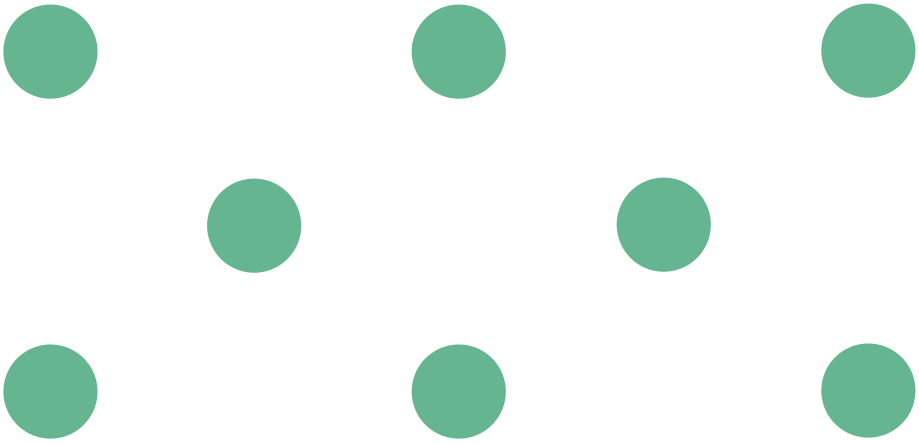
1	2	3
4	5	6
7	8	9

5a. Talk Alouds: Number Talks

Number Talks involve students using mathematical language, rather than just the teacher using the words. This progresses them from just hearing the words (word list, children's literature) to using them.

Maths talks can be used as a whole class discussion or in small groups, but importantly, teachers must encourage maths talk.





How did you do it? (younger student example)

Ask students: 'Without counting the spots, how many do you see? How did you do it?'

Answer: 8 spots.

Possible 'thinking' responses:

- I saw 3, 2, 3 
- I saw 5, 3 
- I saw 3, 2, 3 
- I saw 1, 3, 3, 1 

This example shows students that you can solve maths problems in many different ways.

How did you do it? (older student example)

Teacher gives a calculation orally or in written form

Children perform the calculation

Ask the students "how did you do it?"

e.g. 28 + 37

A student may respond that they added 20 and 30 to get 50 and then 8 + 7 to get 15 and added the two together to get 65.

References:

Humphreys, C. & Parker, R. (2015). ***Making Number Talks Matter.*** Stenhouse: Portland, Maine.

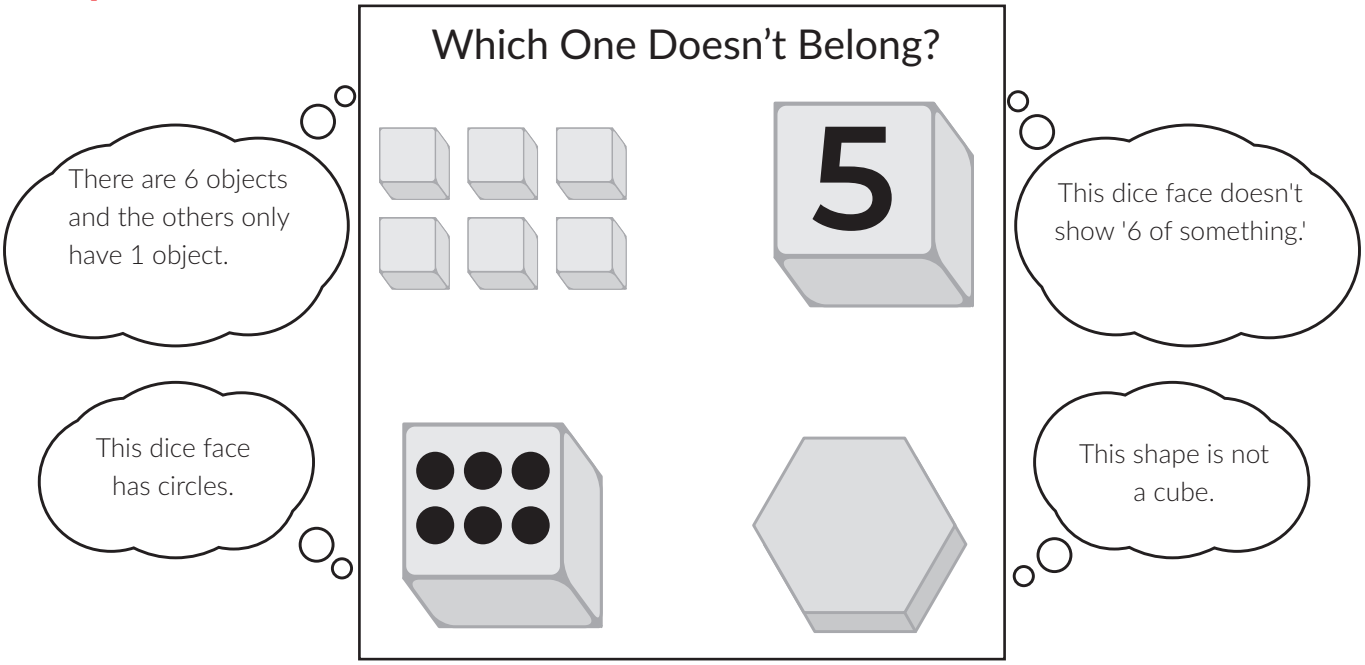
5b. Talk Alouds: Which One Doesn't Belong

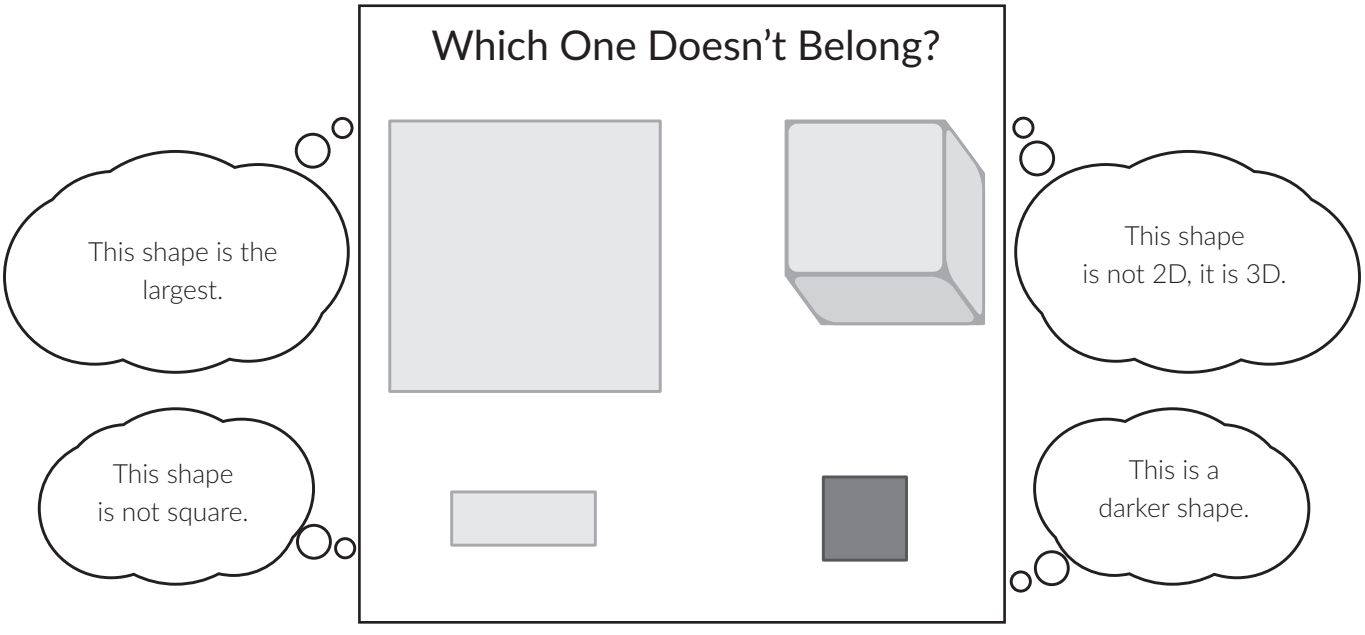
Which One Doesn't Belong? builds on standard number talks by making the student justify their answers to why each of the four choices in the question might not belong to the set. ***Which One Doesn't Belong?*** is an activity that is suitable across many year levels. There is real value in having students design their own ***Which One Doesn't Belong?*** problems.

This idea comes from the book "***Which One Doesn't Belong - A Shapes Book***" by Christopher Danielson. Christopher explains more about the problem type in his youtube video: https://www.youtube.com/watch?v=ACsc1LVyJks

The site wodb.ca has a number of ***Which One Doesn't Belong?*** questions ready to use.

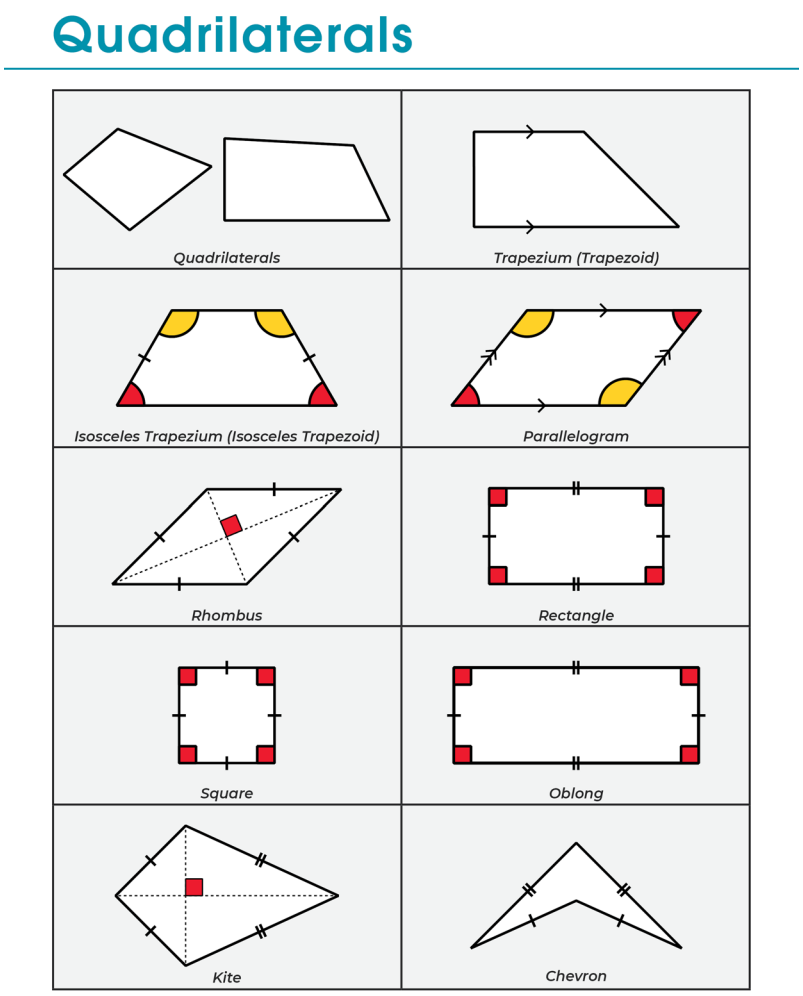
Samples





6a. Word Associations: Posters

Posters may be used to expose students to a variety of related words. Consider a 2D shape (quadrilaterals) poster.



It can highlight the names and features of various 2D shapes. This poster highlights the difference between different quadrilaterals.

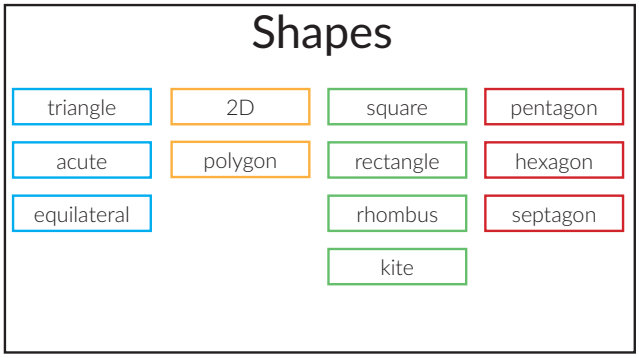
Posters can be used to help introduce mathematics terminology when a topic is introduced. The same poster may then be used to remind students about specific maths terms.

Posters should be removed and replaced on a regular basis - otherwise they run the risk of becoming wallpaper.

6b. Word Associations: Word Walls

The most effective Word Walls are created by the students. Students brainstorm the words related to the topic, write it on a card and pin them to the word wall. Over time the word wall will expand.

A word wall is a dynamic tool for developing mathematical vocabulary.



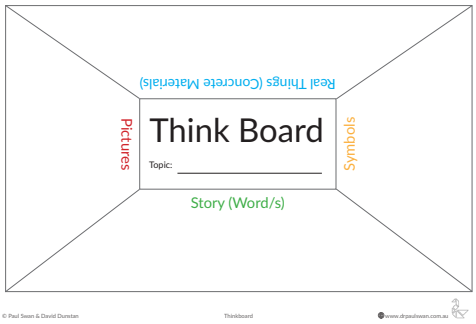
6c. Word Associations: Word Sort

Students are given a set of words - often related in some way - to sort.

Here is a sample set of mixed words.

Perimeter	Circumference
Hexagon	Octagon
Hexahedron	Volume
Billion	Trillion
Triangle	Base
Cent	Century
Percent	Centimetre
Factor	Multiple
Cube	Diameter
Kilogram	Kilolitre
Symmetry	Reflection
Minute	Oblong
Square	Rectangle
Pentomino	

7a. Graphic Organisers: Think Board



A Think Board is a graphic organiser or tool to assist students to make connections.

The more connections that are made:

- the better the understanding of the mathematics
- the more likely it will be that the mathematical knowledge is retained
- and easier it should be to apply the mathematics

The focus of the think board is on finding the connections between the different ways of representing a problem.

Description

Haylock (1984) first described the Think Board. The board is divided into four sections. Each section displays different representations of the same mathematical idea. The various forms of representation include:

- **Stories:** words/vocabulary based on student experience.
- **Real Things:** concrete materials or mathematics manipulatives.
- **Pictures:** Drawings and diagrams (labelled drawing, might include numbers).
- **Symbols:** (number sentence/ equation). (First Steps in Mathematics: Number-Book , 2013, p 87)

How to use

- Arrange students into groups of four, placed around the Think Board's four sections.
- The teacher provides one section of the board and students fill in the other three.
- You can target the section in which your students need more practise:
    - if your students need more experience with word questions, provide them with the equation (goes in the **Symbols** section) and have them write an appropriate **Story.**

Example

- **Teacher's Prompt: 7 + 3 = 10 (Symbols)**
- Give students time to discuss the idea with their neighbours, brainstorming what might fit in each section of the Think Board before deciding on what to record.

Students will need to fill in the sections:

- **Stories:** An appropriate story will need to be written in, e.g. "I had 7 red apples and I was given 3 green apples. I now have 10 apples"
- **Real Things:** Place appropriate concrete materials or mathematics manipulatives, e.g. 7 red counters and 3 green counters.
- **Pictures:** Students create drawings and diagrams. Example drawings could include a ten frame with 7 and 3 in different colours or a number line showing jumps of 7 and 3. Make sure the drawing matches the story. A drawing of 10 uncoloured apples does not necessarily accurately describe the story.
- **Symbols:** Students write in the symbols associated with the prompt. In this case this section was provided by the teacher, but this will not always be the case.

The Think Board can be used as a whole class, in groups of four or individually.

- Starter cards have been provided on the next pages which can be used as topic starters.

Real Things (Concrete Materials)

Pictures

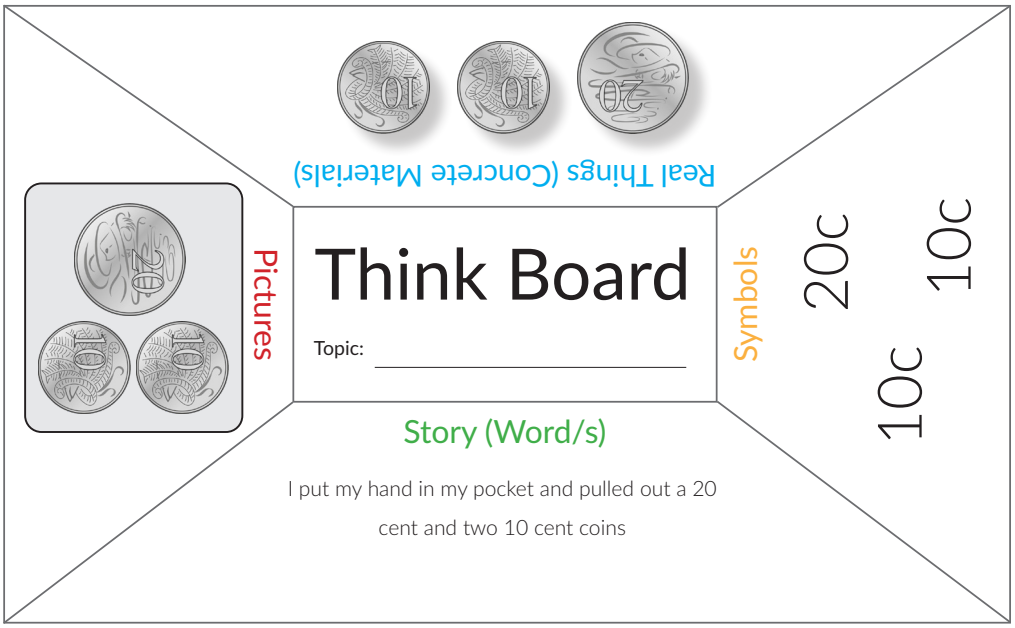
Think Board

Topic: _____

Symbols

Story (Word/s)

Think Board Sample

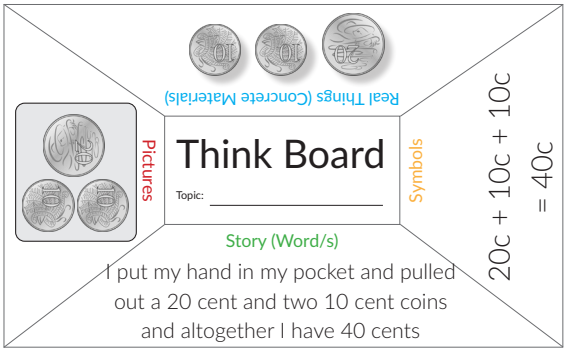


Example 1: Recognising Coins (Year 1)

- **Stories:** e.g. a student might write (or say, in the case of young children) something like the following: "I put my hand in my pocket and pulled out a 20 cent and two 10 cent coins."
- **Real Things:** the student puts physical materials (real / plastic) of the same coins as the prompt card on the board.
- **Pictures:** Draw a representation or, if using the prompt cards, place it here. Note: Coins have a symbol (the number 20) on them.
- **Symbols:** 20c, 10c

Example 2: Counting Coins (Year 2)

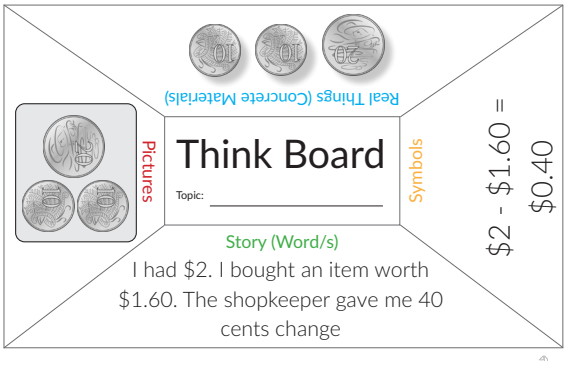
- **Stories:** e.g. a student might write something like the following: "I put my hand in my pocket and pulled out a 20 cent and two 10 cent coins and altogether I have 40 cents."
- **Real Things:** same as example 1
- **Pictures:** same as example 1
- **Symbols:** Student writes the equation in the section: 20c + 10c + 10c = 40c



Example 3: Change

The teacher explains that the picture card showing a total of 40c represents the change given in a transaction.

- **Stories:** e.g. a student might write something like the following: "I had $2. I bought an item worth $1.60. The shopkeeper gave me 40 cents change."
- **Real Things:** same as example 1
- **Pictures:** same as example 1
- **Symbols:** $2.00 - $1.60 = $0.40



Assessment

- Think Boards promote discussion – look for mathematical language used.
- Assess individual student's understanding of a mathematical idea – look at all sections of the Think Board individually to find strengths and weaknesses. (Herrington, 1988)

References & Further Reading

Department of Education WA. (2013). ***First Steps in Mathematics: Number-Book 2***. Perth: D.O.E.

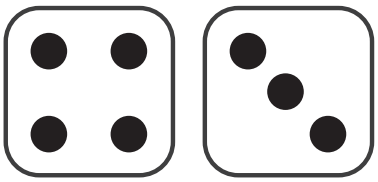
Haylock, D.W. (1984). ***A mathematical think board***. Mathematics Teaching, 108, 4-5.

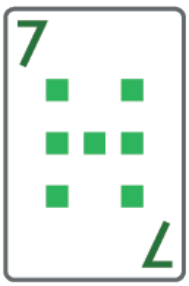
Herrington, A.J. (1988). ***Mathematical Understanding and the Think Board***. Paper presented at the Annual Conference of the Mathematical Association of the Western Australia, Muresk.

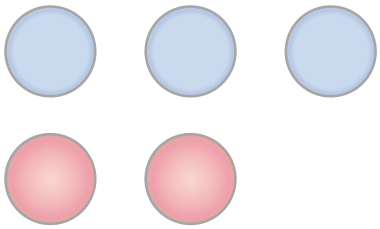
Story Prompt Cards

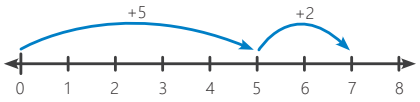
I have 3 coins that total 40c	I rolled two dice. The total was 7.	We played Snap: one more one less. I need a 6 or an 8.
I had 3 blue stickers and my friend gave me 2 red stickers. I now have 5 stickers.	I did a big jump and landed on 5, then I did a small jump and laned on 7.	I jumped backwards 2 places from 6.
I was on 3 and rolled 2, so I skipped the number 4.	I had 6 balloons. One burst.	David added two counters to his ten frame and filled it up.
I had a score of 8 and then scored 7. I only have to score 5 more to reach 20.	I had 28 popsticks but I lost 5 of them.	When we measured our heights Lucy was the tallest and Harper was the shortest.

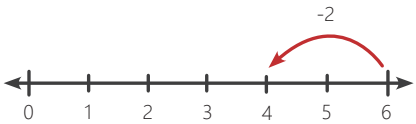
Picture Prompt Cards

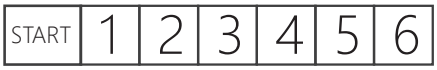


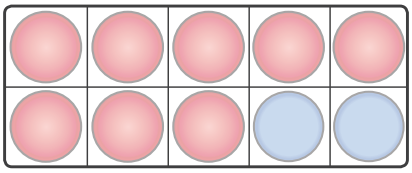


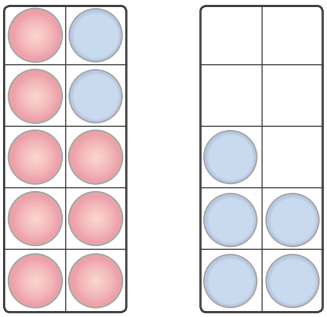


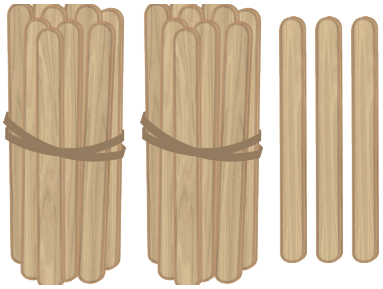


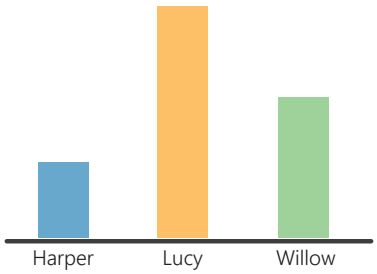


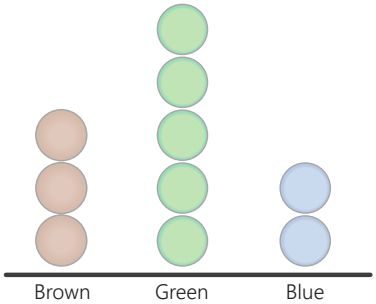


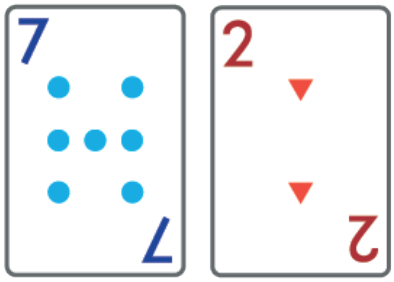


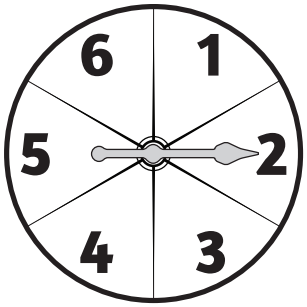


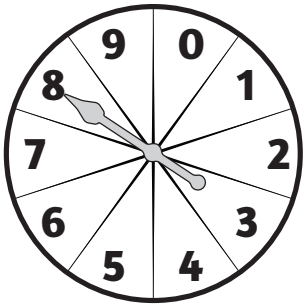


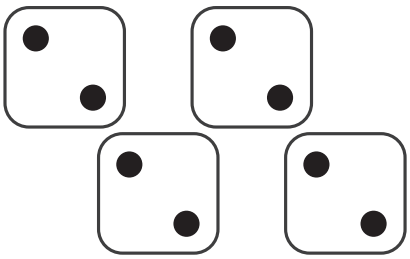








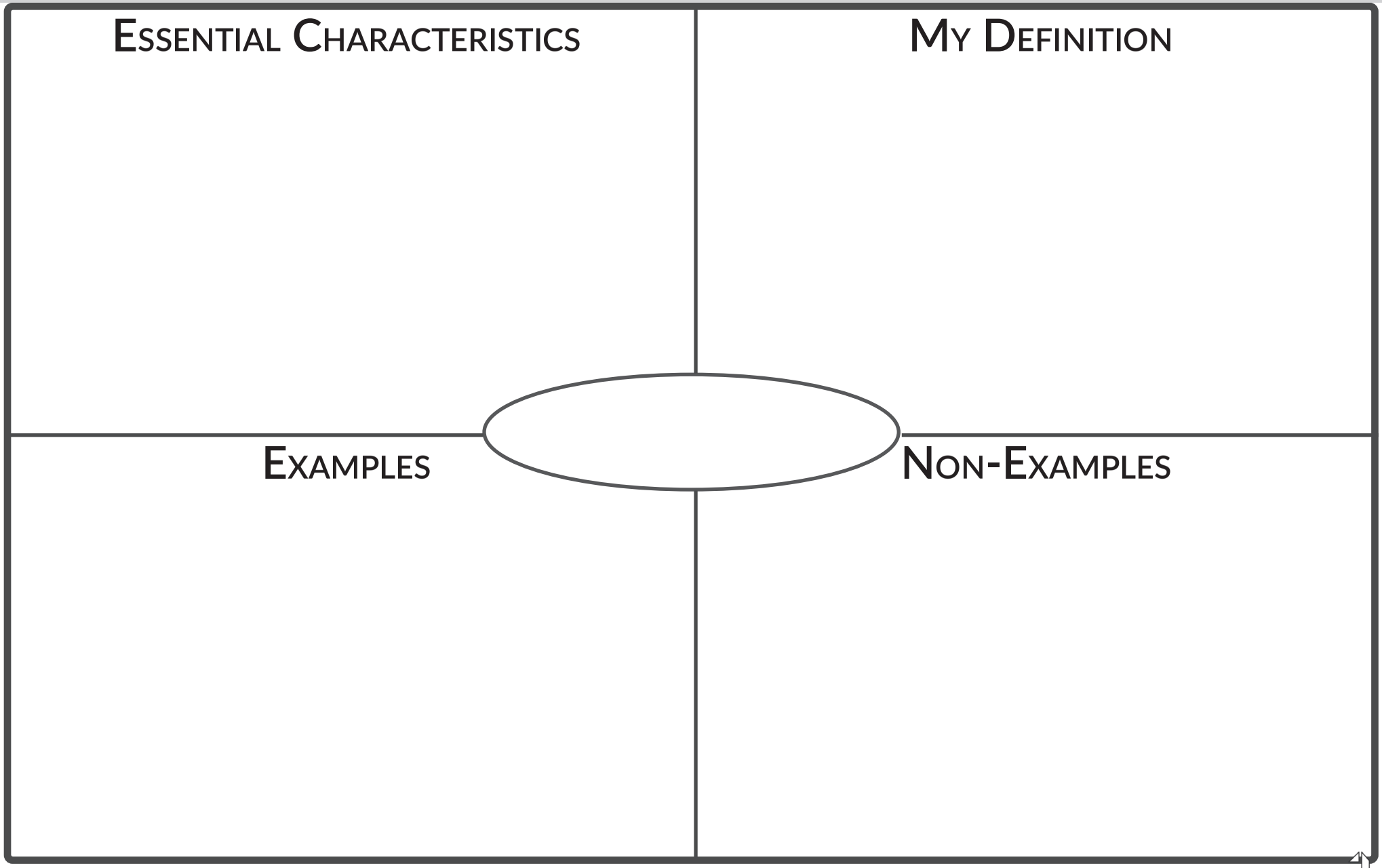




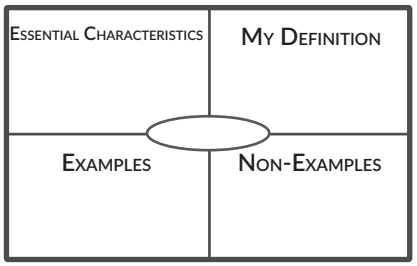
Symbol Prompt Cards

3 + 2 = 5	__ + __ = 7	6, 7, 8
10c + 10c + 20c	5 + __ = 7	6 - 2 = __
3, 4, 5	6 - 1 = __	__ + 2 = 10
8 + 7 = __ __ + 5 = 20	28 - 5 = __	Lucy > Harper
mode = green	50c + 50c = $1 50c + 20c + 20c + 10c = $1 50c + 20c + 10c + 10c + 10c = $1 ...	__ + __ = 9
even + even = even	__ - 6 = 2 6 + 2 = __	4 lots of 2 = 8 4 x 2 = 8

Frayer Board



7b. Graphic Organisers: Frayer Board



The Frayer Board is a graphic organiser or tool to assist students describe and explain new terms and concepts. It is divided into four parts.

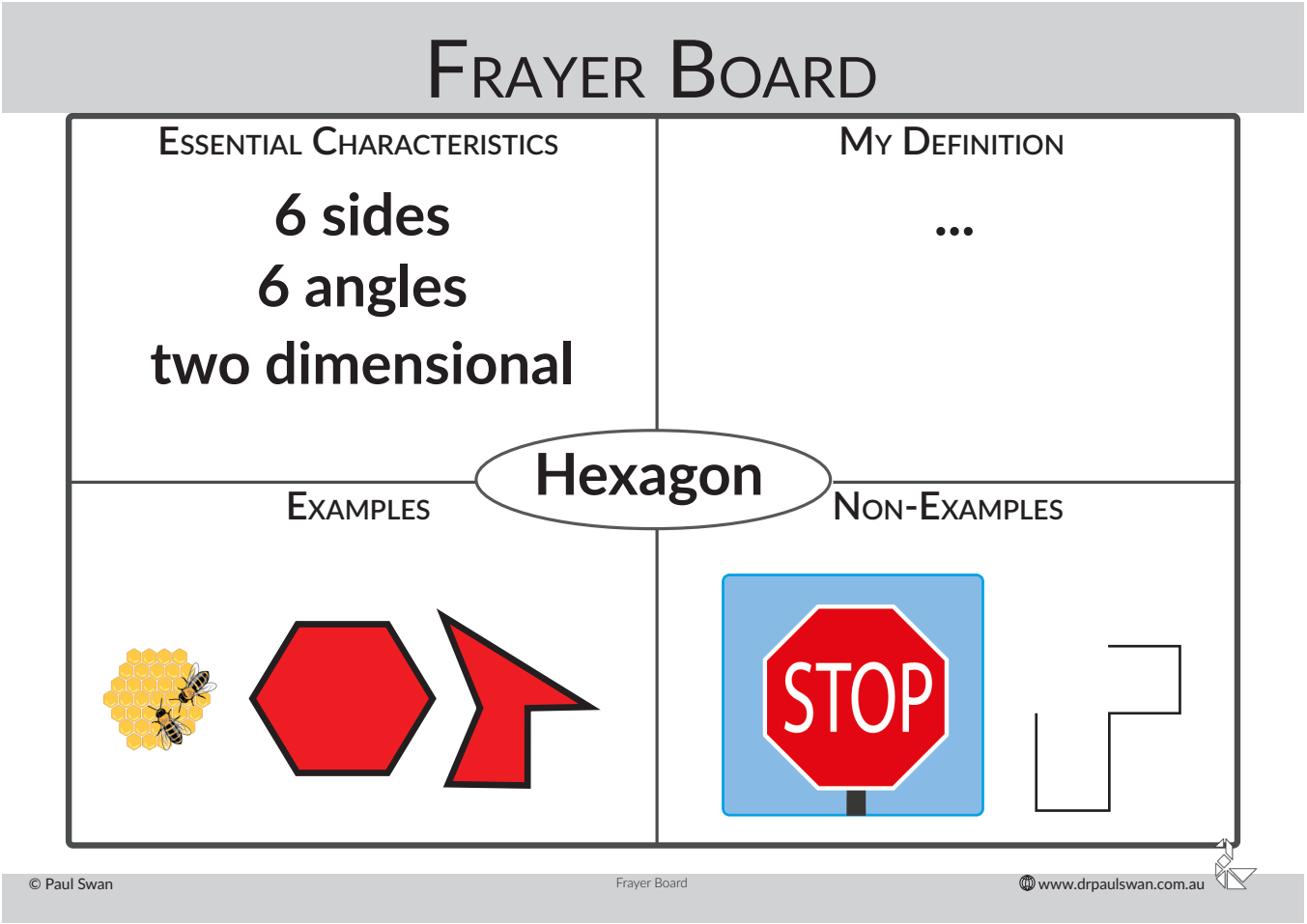
How to use a Frayer Board

- The Frayer Board may be used as a whole class, groups of four or individually.
- Provide students the word or phrase.
- This topic word goes in the centre oval.

Students fill in the sections related to the topic word:

- **1. Essential Characteristics:** Common descriptors associated with the topic word
- **2. Examples:** examples of the topic word (pictures / drawings etc are suitable here)
- **3. Non-Examples:** items that do not fully meet the essential characteristics
- **4. My Definition:** based on the other 3 sections, students attempt to make a personal definition of the topic word.

Sample Frayer Board - Topic Word: Hexagon



Extending Frayer Board Activities

- **Missing Word:** Provide the students with a completed Frayer Board with the centre word removed and ask them to decide on what word is missing.
- **Before and After:** Students fill in a frayer board at the start of a lesson as a tune-in activity. After the lesson, students add to the frayer board with their new words as a reflection activity.
- **Check the Dictionary:** Once they have written their 'my definition', students consult a Mathematics Dictionary and compare the actual definition with their own.

Assessment

- Assess individual student's understanding of a mathematical idea – look at all sections of the Frayer Board individually to find strengths and weaknesses.
- Ask students to complete a Frayer Board prior to starting a unit of work and then again at the end and note any growth. Students can use one colour pen initially and then change colour when revisiting the board.

References

Frayer Model adapted from Frayer, D. A., Frederick, W. C., & Klausmeier, H. G. (1969). ***A schema for testing the level of concept mastery*** (Technical report No. 16). Madison, WI: University of Wisconsin Research and Development Center for Cognitive Learning.

7c. Graphic Organisers: Knowledge Rating Scale

The Knowledge Rating Scale assesses their knowledge and understanding of vocabulary. Place a sheet in between a group of four students and ask about a set of words. Students will mark from 'never heard of it' to 'I can describe/define it.'

Knowledge Rating Scale

Word	Have no clue about it	I have seen or heard it	I have a fairly good idea of what it means	I can define it. I can explain what it means